Self-potential and induced polarization:
Geophysical tools to map flowpaths and monitor contaminant plumes

Presentation LAN

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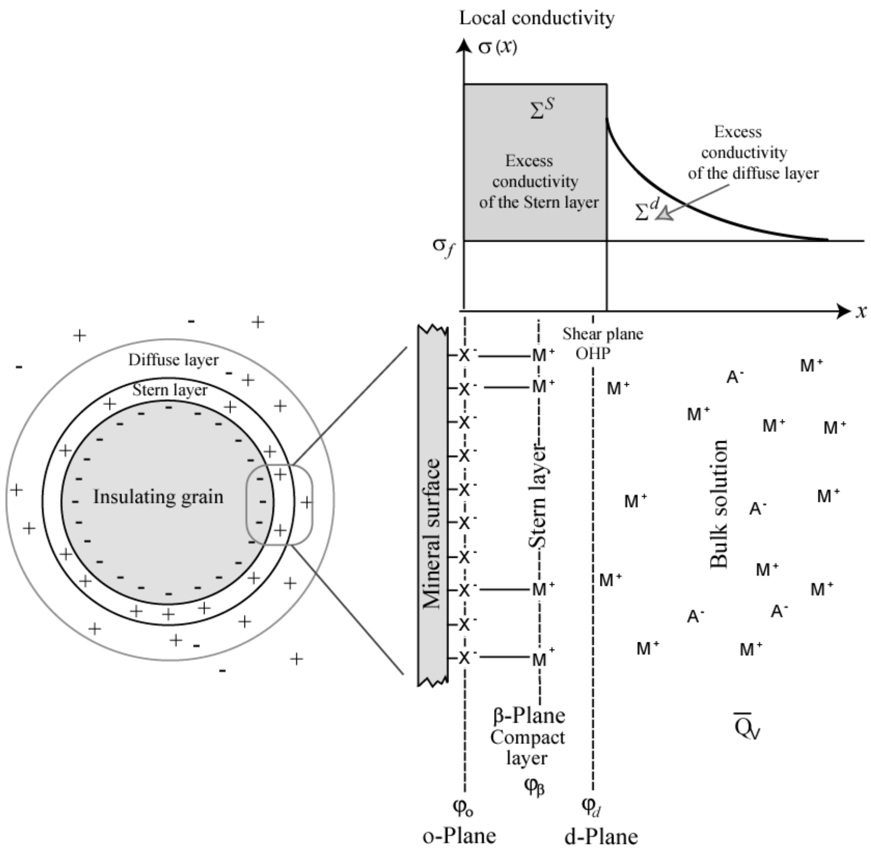
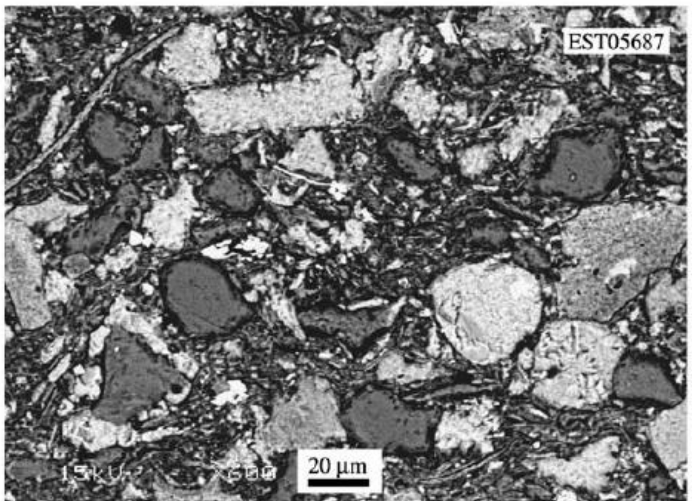
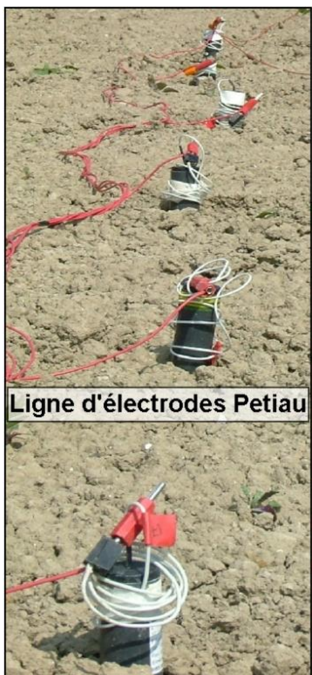
CNRS-UMR6143 M2C





Medical Science (electro-encephalography)

PART I. Self-potential



Network of non-polarizing electrodes connected to a voltmeter

Geophysics (self-potential)

"M"-form of the transport equations

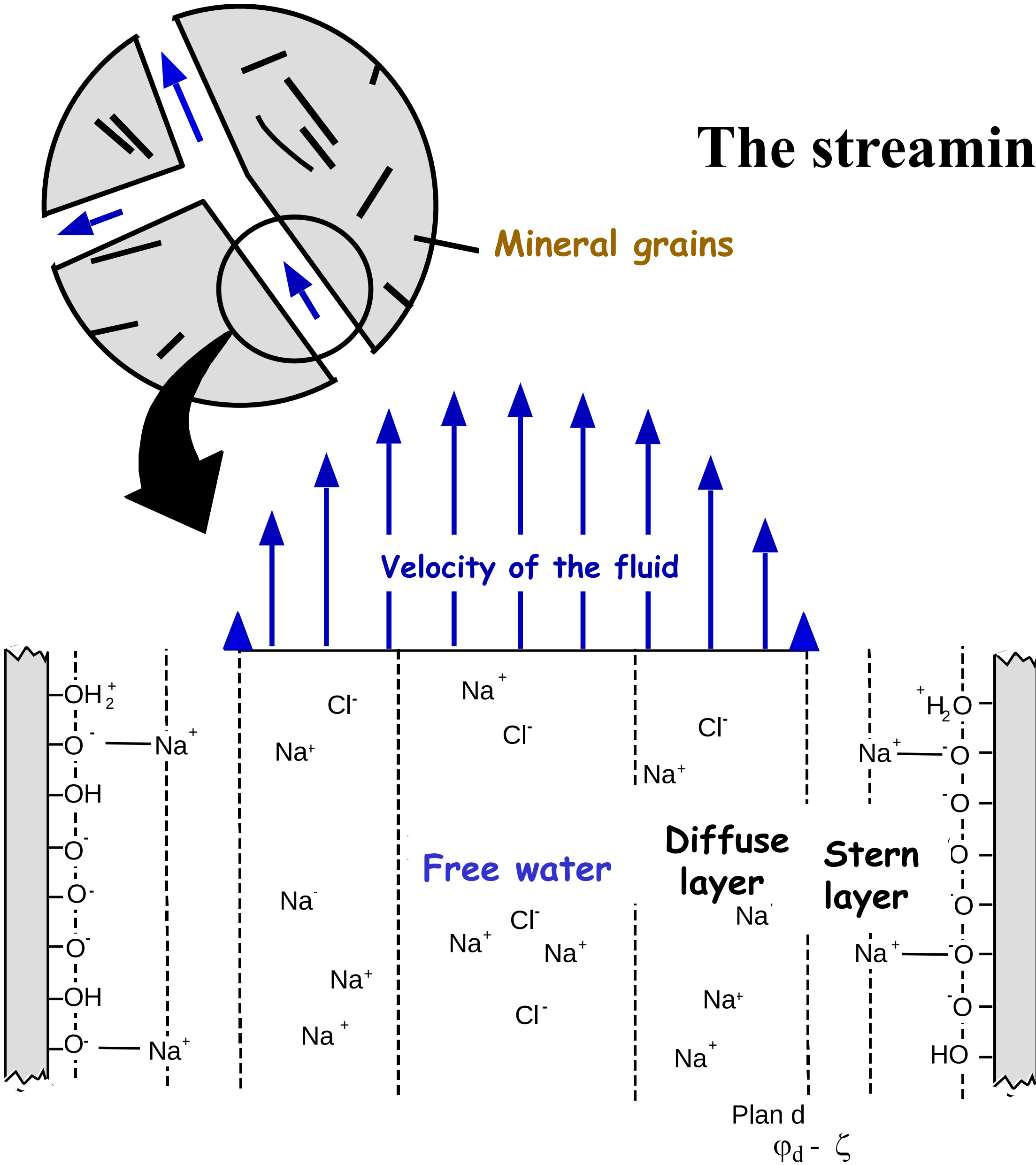
Generalized constitutive equations

Take-home message: any king of non-equilibrum disturbance generate an EM signal

"M"-form	Chemical potential	Electric potential	Fluid pressure	Temperature
Salt flux $\mathbf{J}_d$	**Fick's law**	**Electromigration**	**Convective diffusion**	**Soret effect**
Current density $\mathbf{J}$	**Diffusion current**	$\boxed{\textbf{Ohm's law}}$	$\boxed{\textbf{Electrofiltration}}$	**Thermo-electricity**
Darcy velocity $\mathbf{u}$	**Osmosis**	$\boxed{\textbf{Electro-osmosis}}$	$\boxed{\textbf{Darcy's law}}$	**Thermo-osmosis**
Heat flux $\mathbf{H}$	**Dufour effect**	**Peltier effect**	**Convective flux**	**Fourier's law**

Electroosmosis (Reuss, 1805), Darcy's law (1856), the streaming potential (Quincke, 1859)

The streaming current



The flow of ground water generates an electrical current density

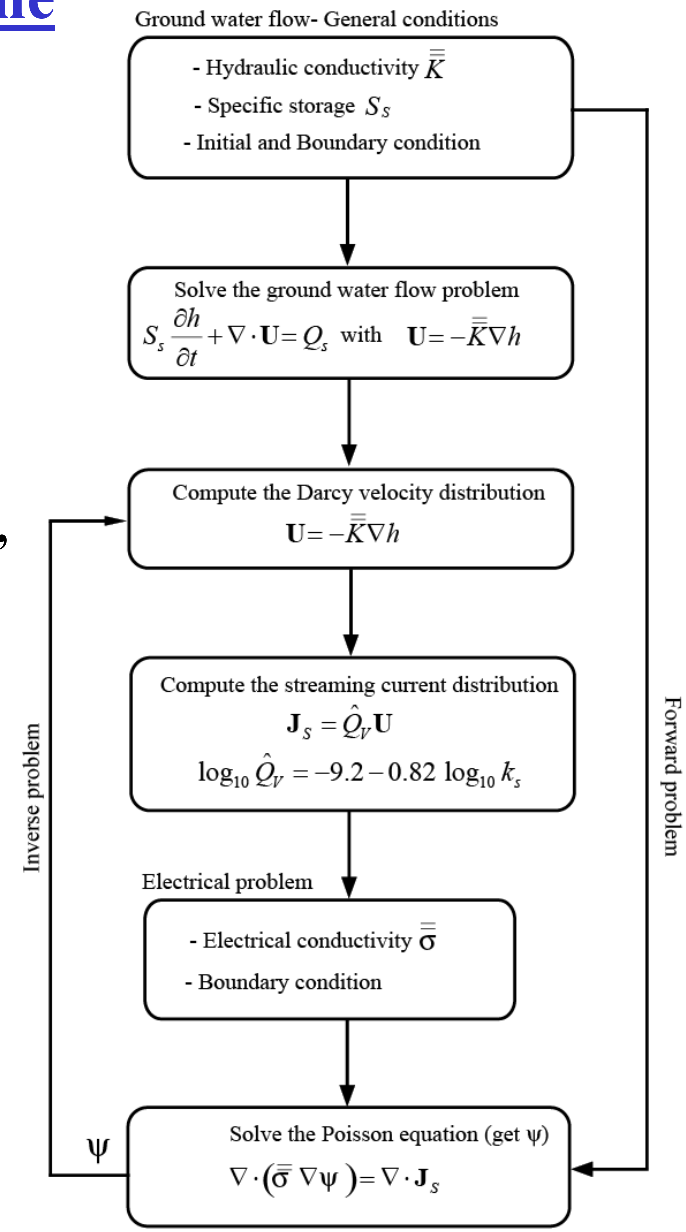
Forward and inverse problems of the self potentiel signal

h: hydraulic head (hydraulic Pressure)

ψ : Electric potentiel

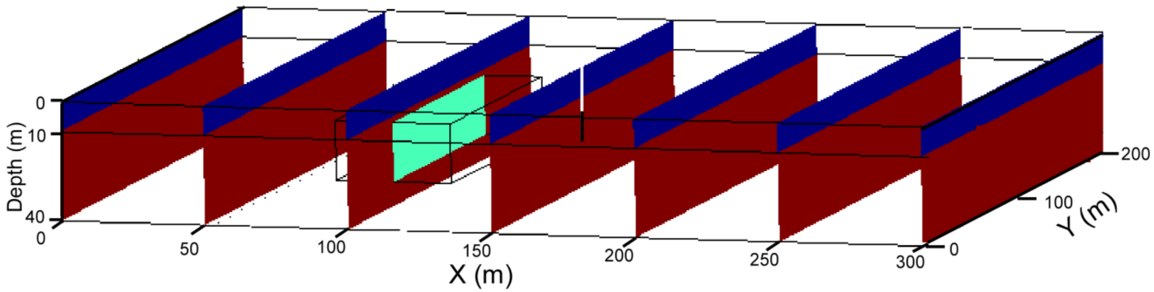
Forward problem: **Finite Element, Finite Difference or Finite volume.**

Inverse problem: **Gradient based methods, Stochastic methods**

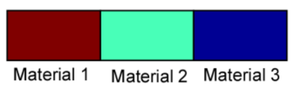


Application to pumping tests

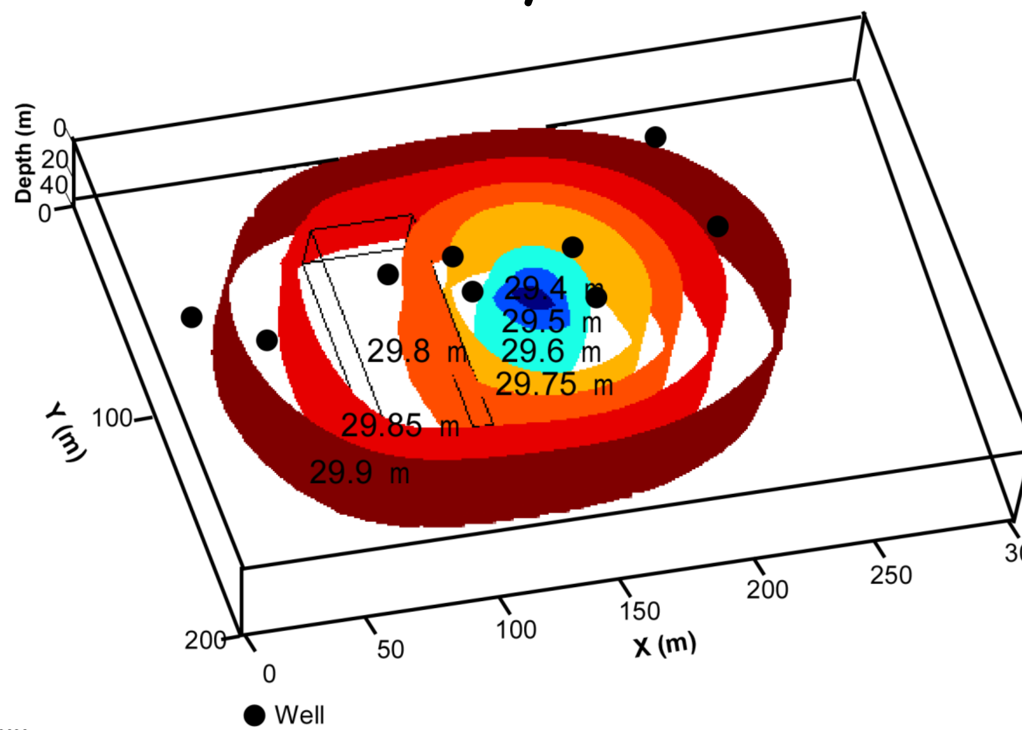
Numerical modeling



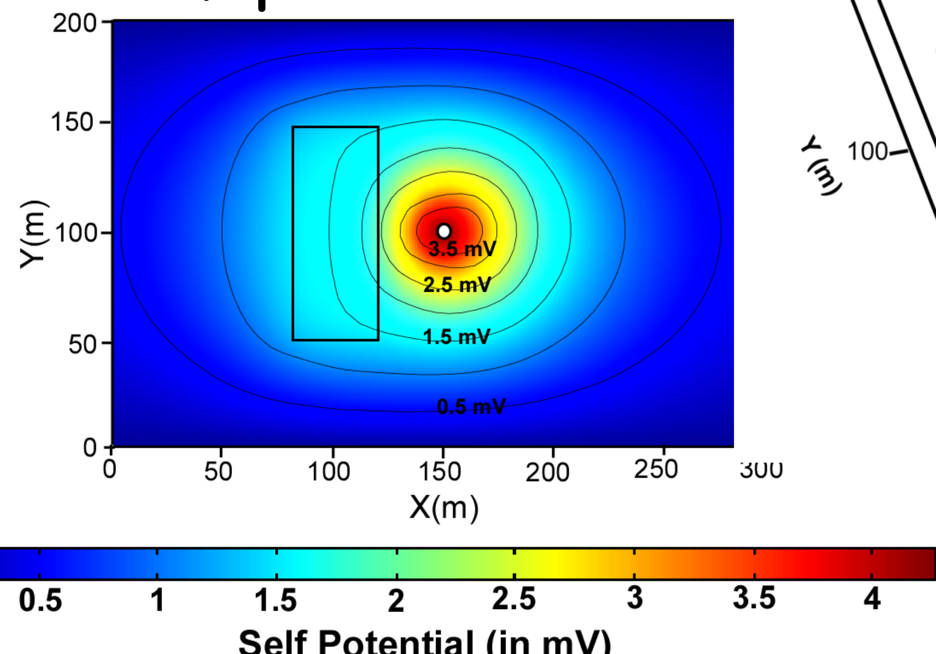
model



Hydraulic heads



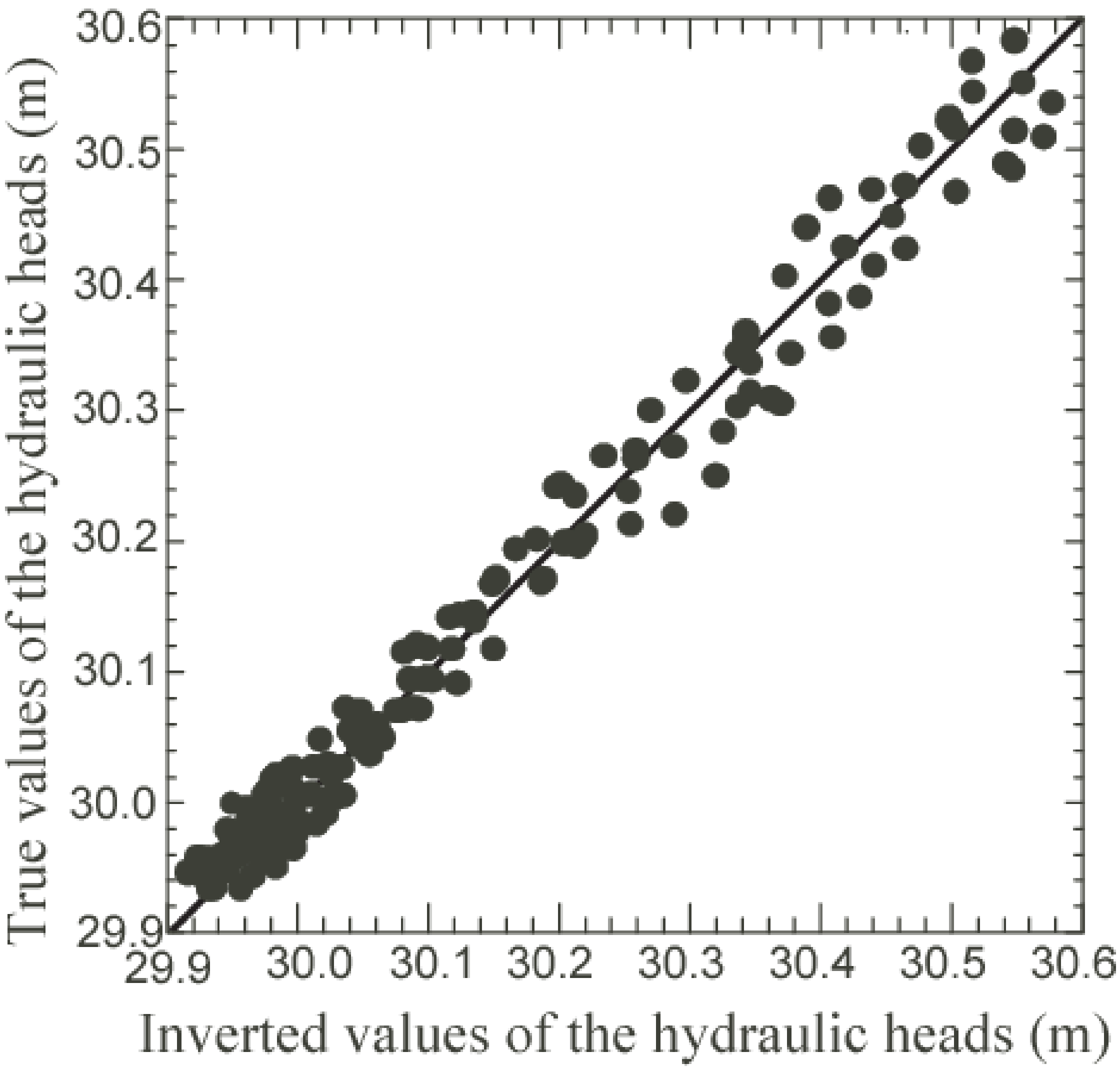
Self-potential

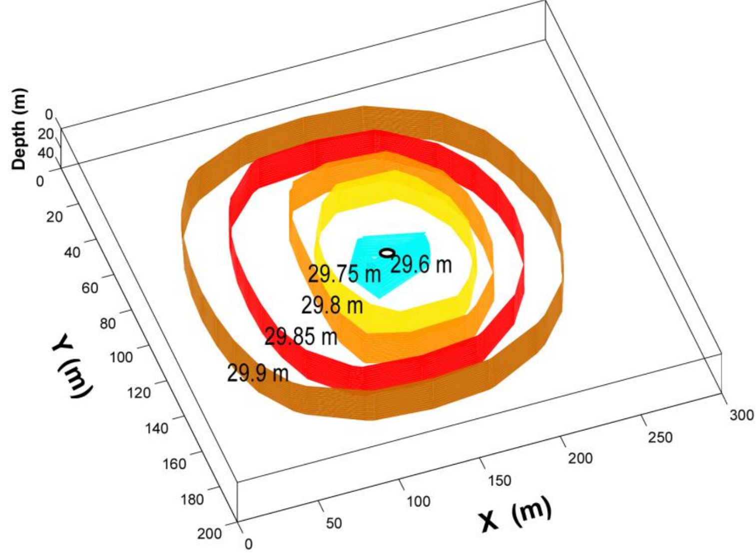


Solution of the inverse problem

the posterior probability density (Bayesian Approach)

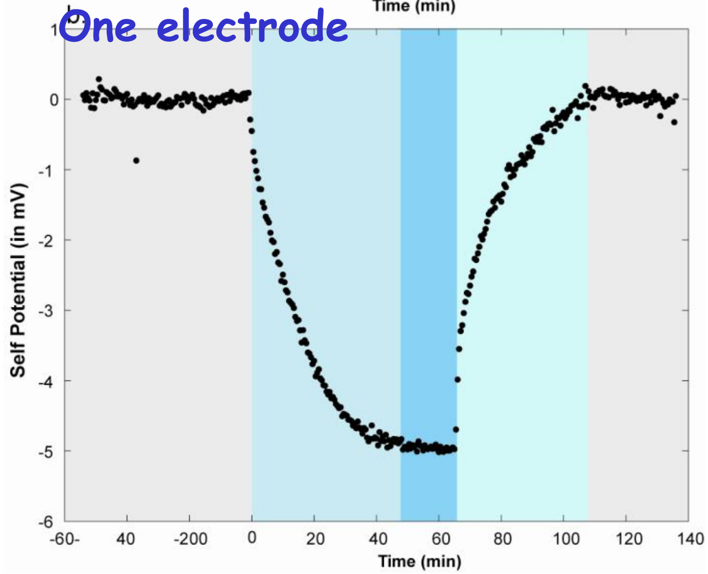
$$\sigma(\bar{h}) \propto \exp\left[-\frac{1}{2}\left(\bar{\bar{K}}\,\bar{h}-\bar{d}_{obs}\right)^{T}\bar{\bar{C}}_{d}^{-1}\left(\bar{\bar{K}}\,\bar{h}-\bar{d}_{obs}\right)-\frac{1}{2}\left(\bar{h}-\bar{h}_{prior}\right)^{T}\bar{\bar{C}}_{m}^{-1}\left(\bar{h}-\bar{h}_{prior}\right)\right]$$



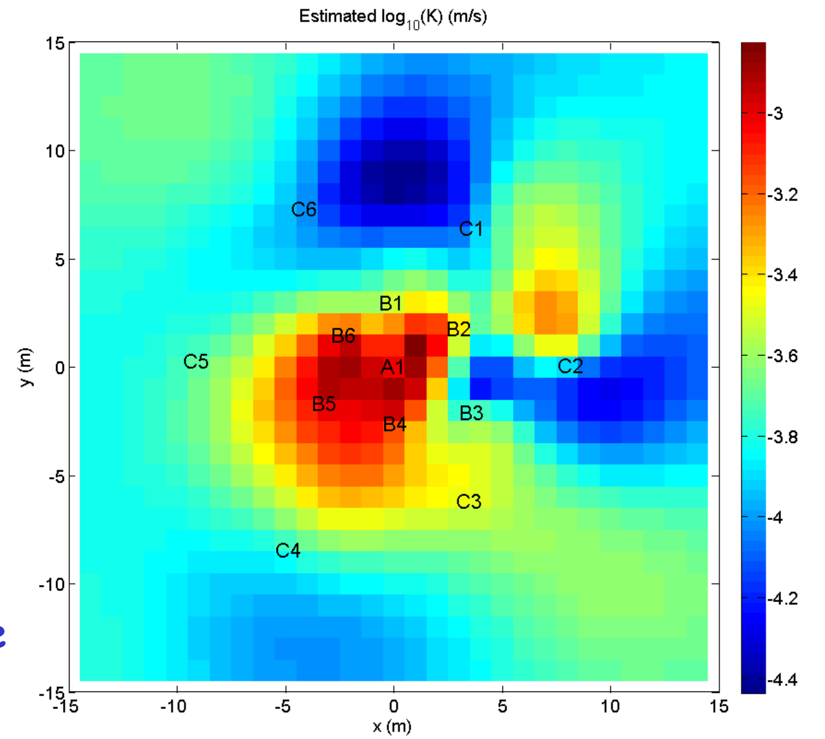


Reconstruction of the water table

One electrode



2 steps Inversion
(Gauss-Newton)



Case study at the Boise hydrogeological test site
(Jardani et al., Ground Water, 2009)

Inversion of self-potential data with Tikhonov regularization

Poisson equation

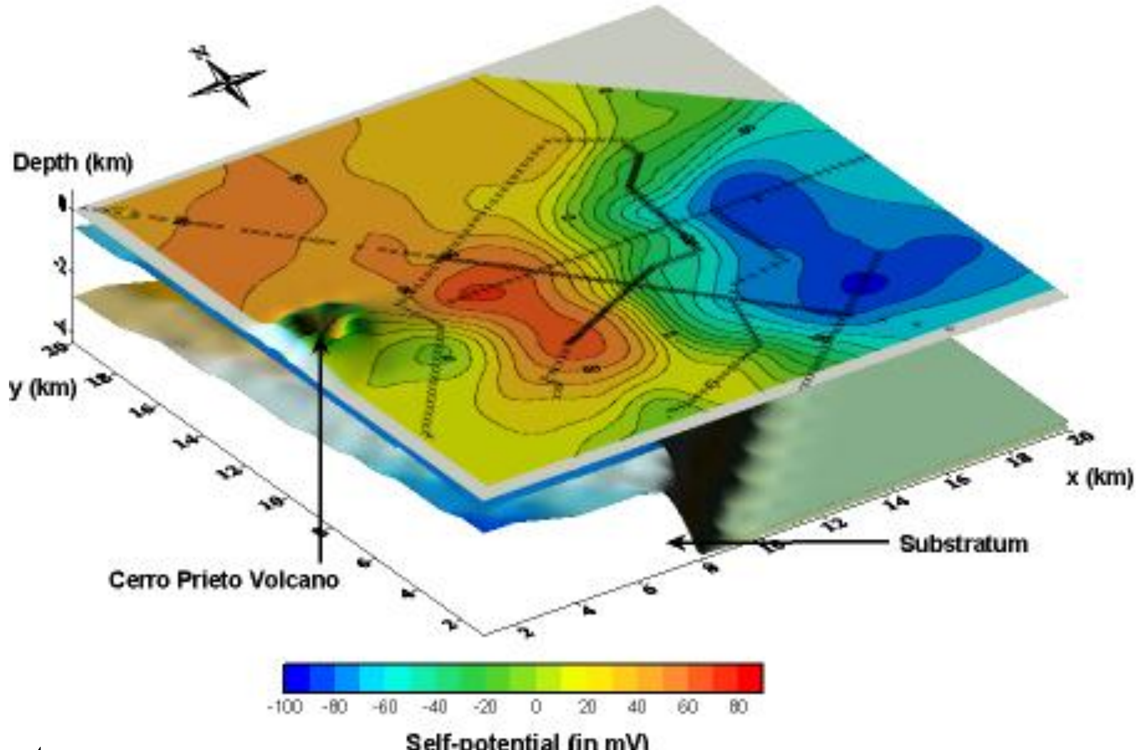
Forward modeling $\nabla\cdot(\sigma\nabla\phi) = \nabla\cdot\mathbf{j}_S$

$\Im$ Source term computed from the Darcy velocity

Inverse modeling

$$\varphi(P) = \int_{\Omega}\mathbf{K}(P,M)\,\mathbf{j}_S(M)\,dV$$

Where K is the kernel



Minimization of a cost function:

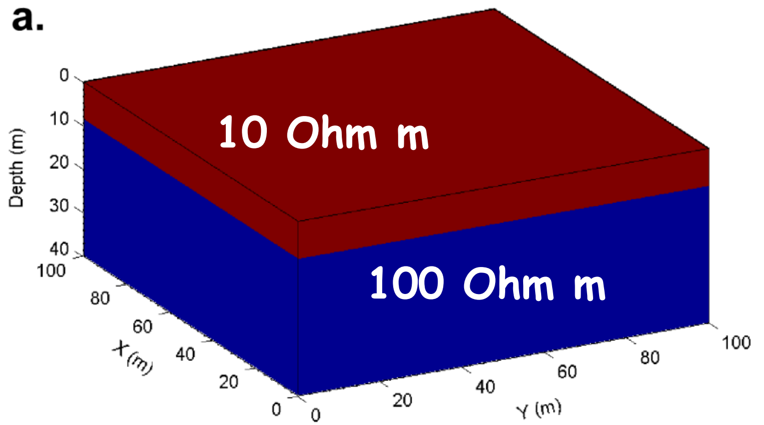
Data misfit   Regularization term

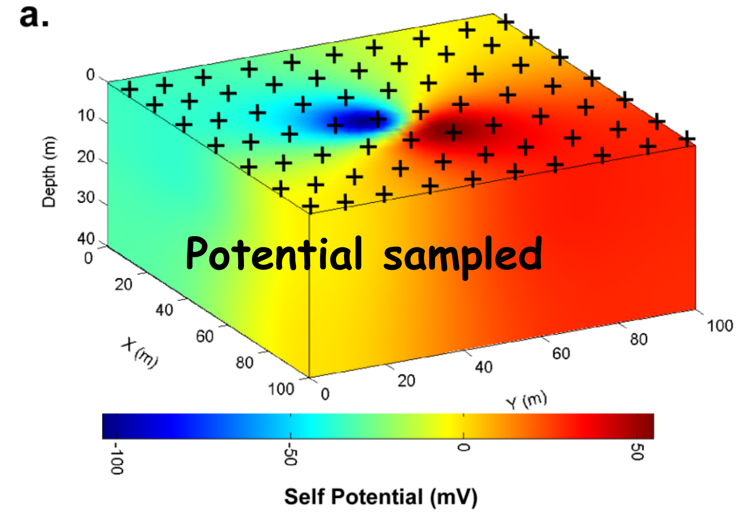
$$\psi = \left\|\mathbf{W}_d(\mathbf{Km}-\varphi_d)\right\|_2 + \lambda\left\|\mathbf{W}_m(\mathbf{m}-\mathbf{m}_0)\right\|_2$$

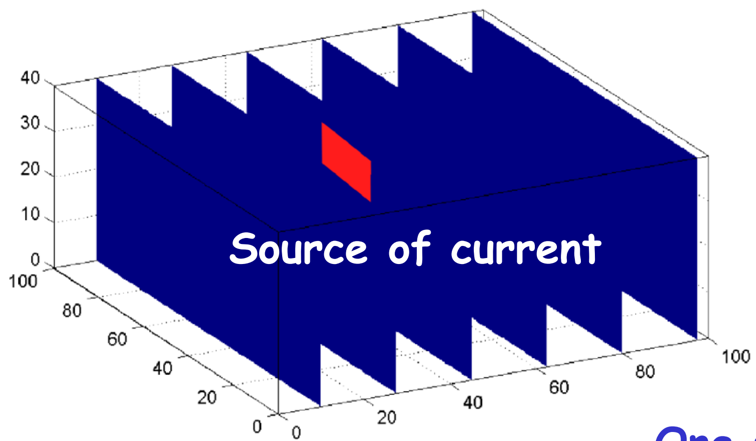
$$\mathbf{m}^* = \left[\mathbf{K}_w^T(\mathbf{W}_d^T\mathbf{W}_d)\mathbf{K}_w + \lambda(\mathbf{W}_m^T\mathbf{W}_m)\right]^{-1}\left[\mathbf{K}_w^T(\mathbf{W}_d^T\mathbf{W}_d)\varphi_d + \lambda(\mathbf{W}_m^T\mathbf{W}_m)\mathbf{m}_0\right]$$

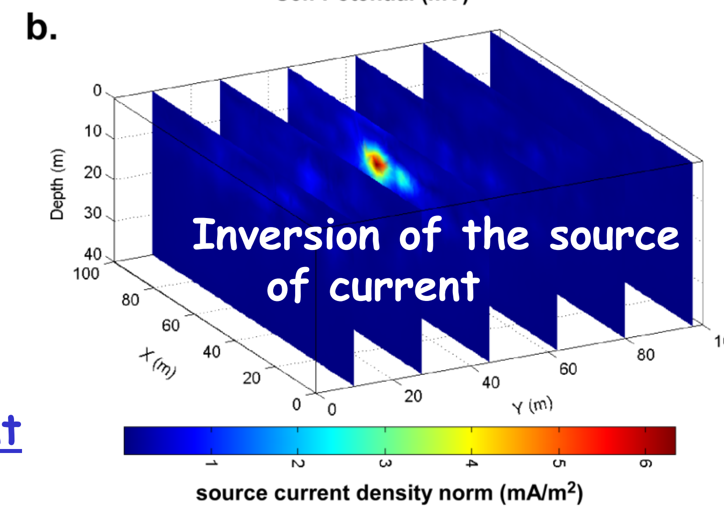
The distribution of the electrical resistivity is taken into account in the Kernel

The regularization coefficient λ is determined by the L-shape method of Hansen (1998)

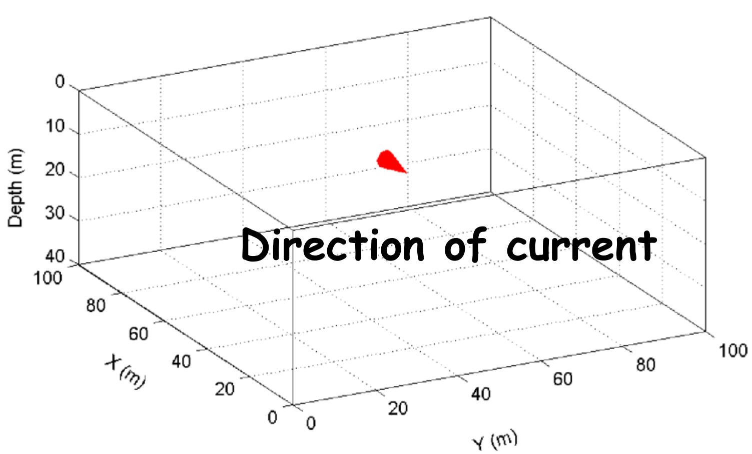


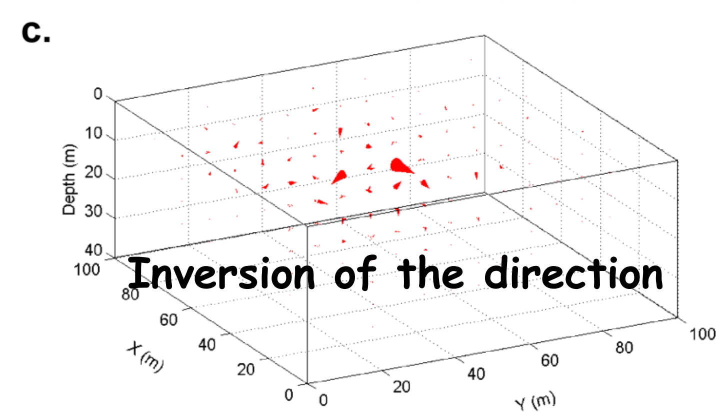




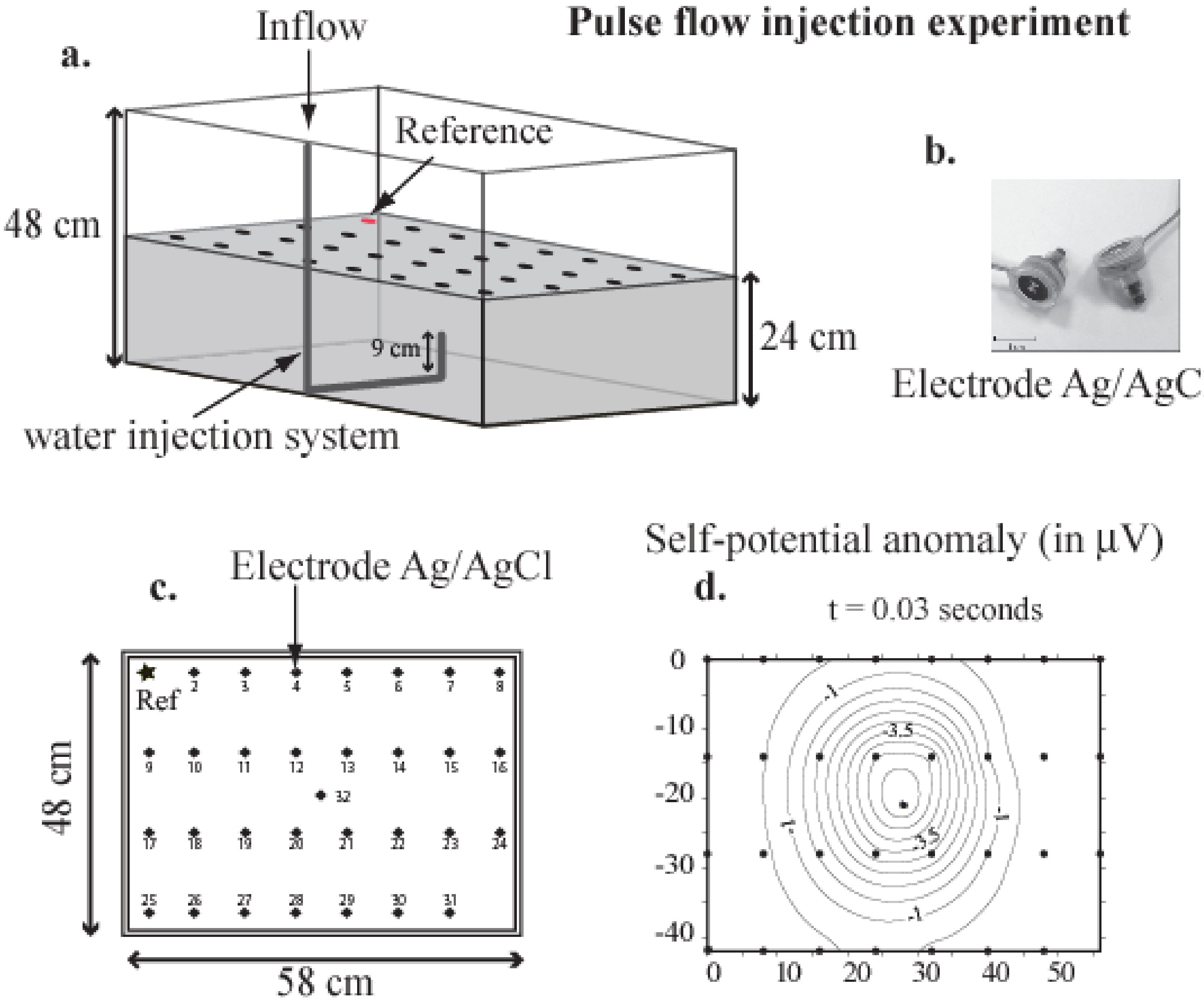


One source of current



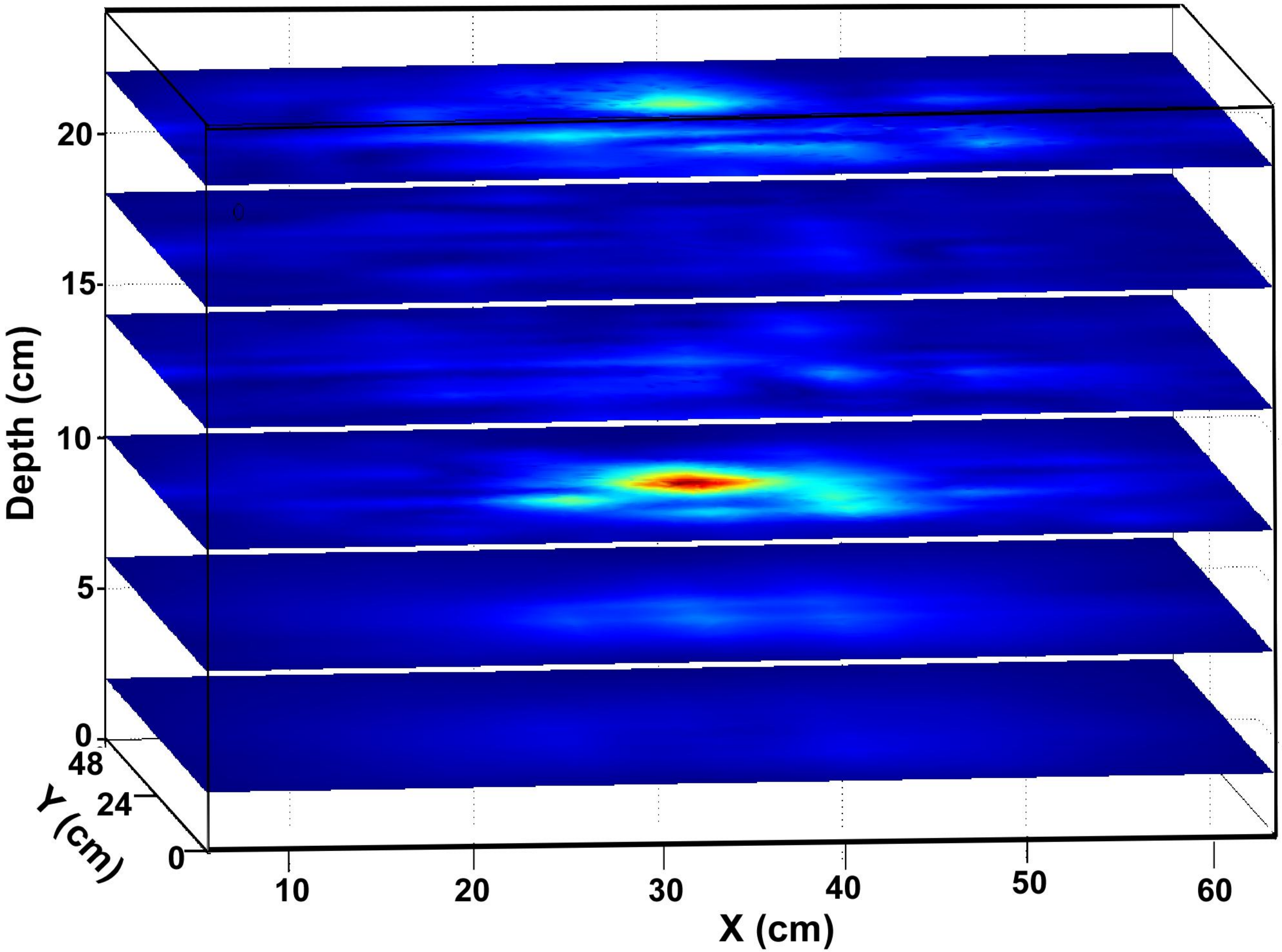


Detection of a hydromechanical source in a sandbox



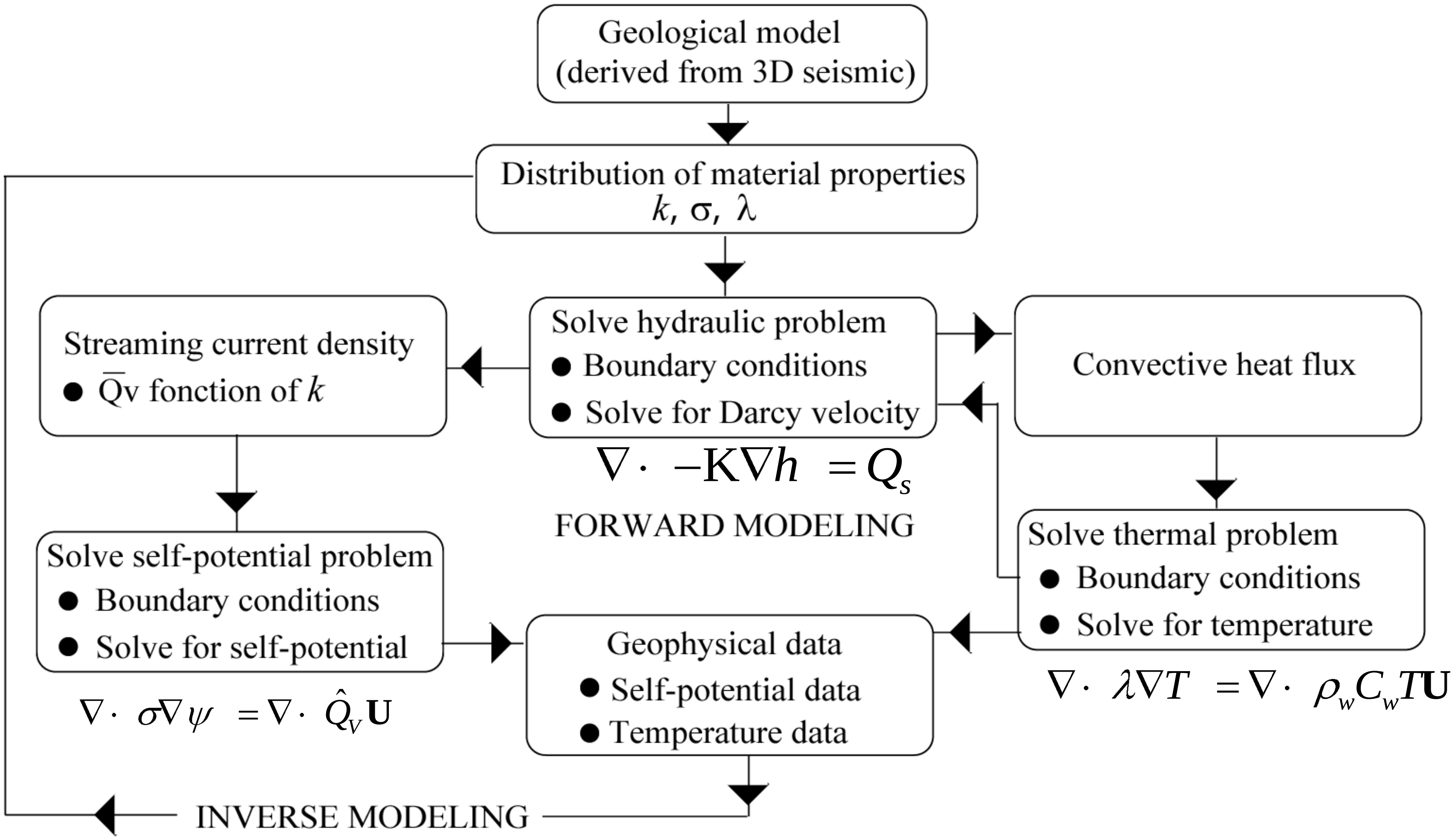
Localisé une source hydromécanique dans un bac à sable

Finding the source current density



The inverted localization of the source agrees with the position of the outlet of the capillary

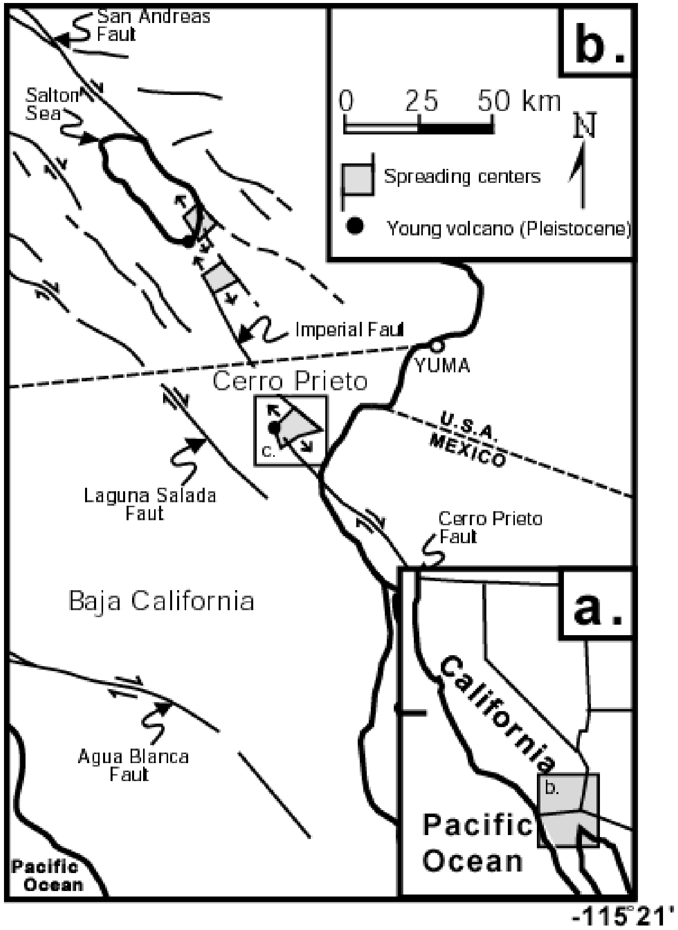
Inverse modeling: going one step further with Markov Chain MC samplers

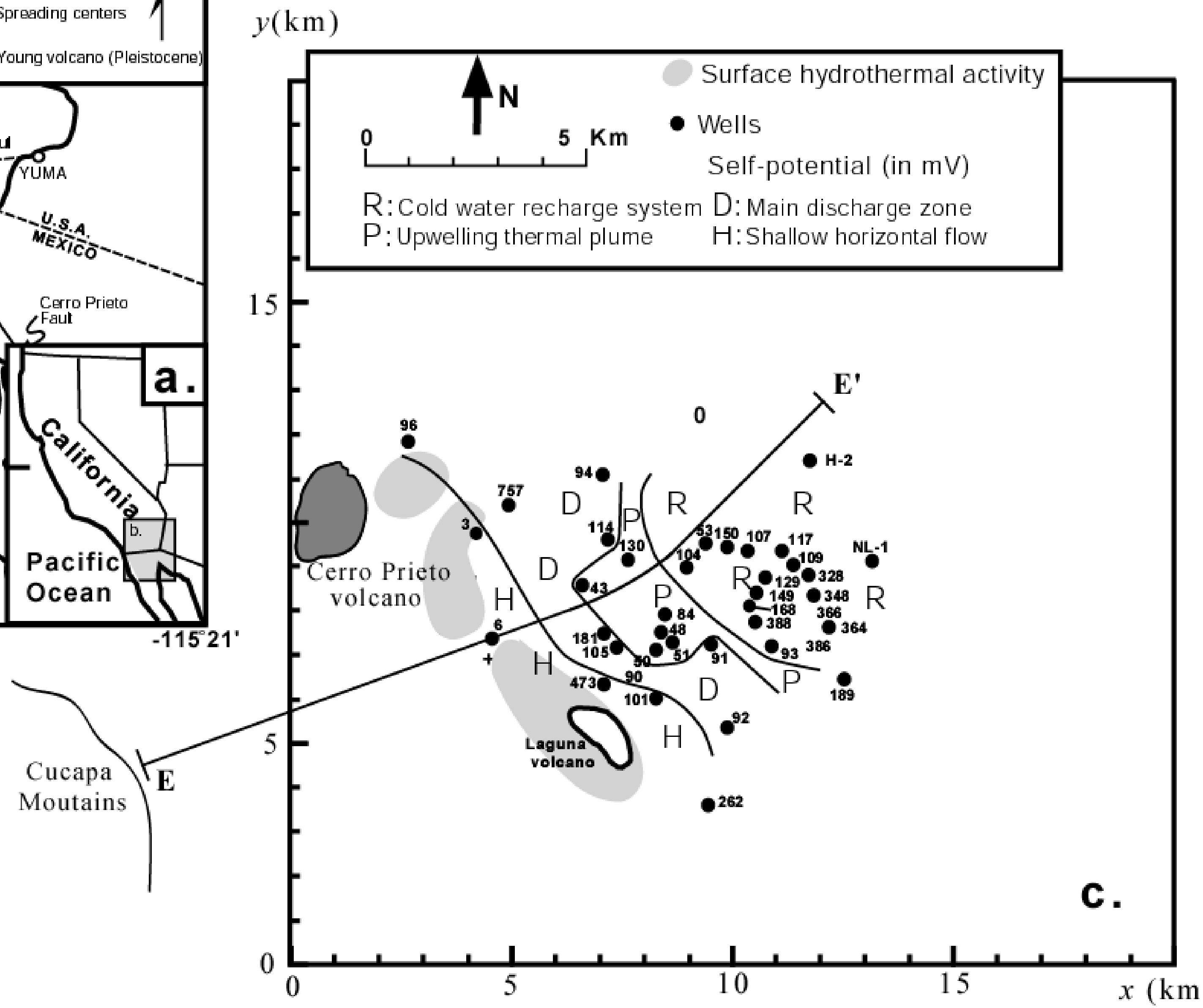


MCMC sampler (Adaptative Metropolis algorithm, AMA, Haario et al., 2001) / Bayesian framework

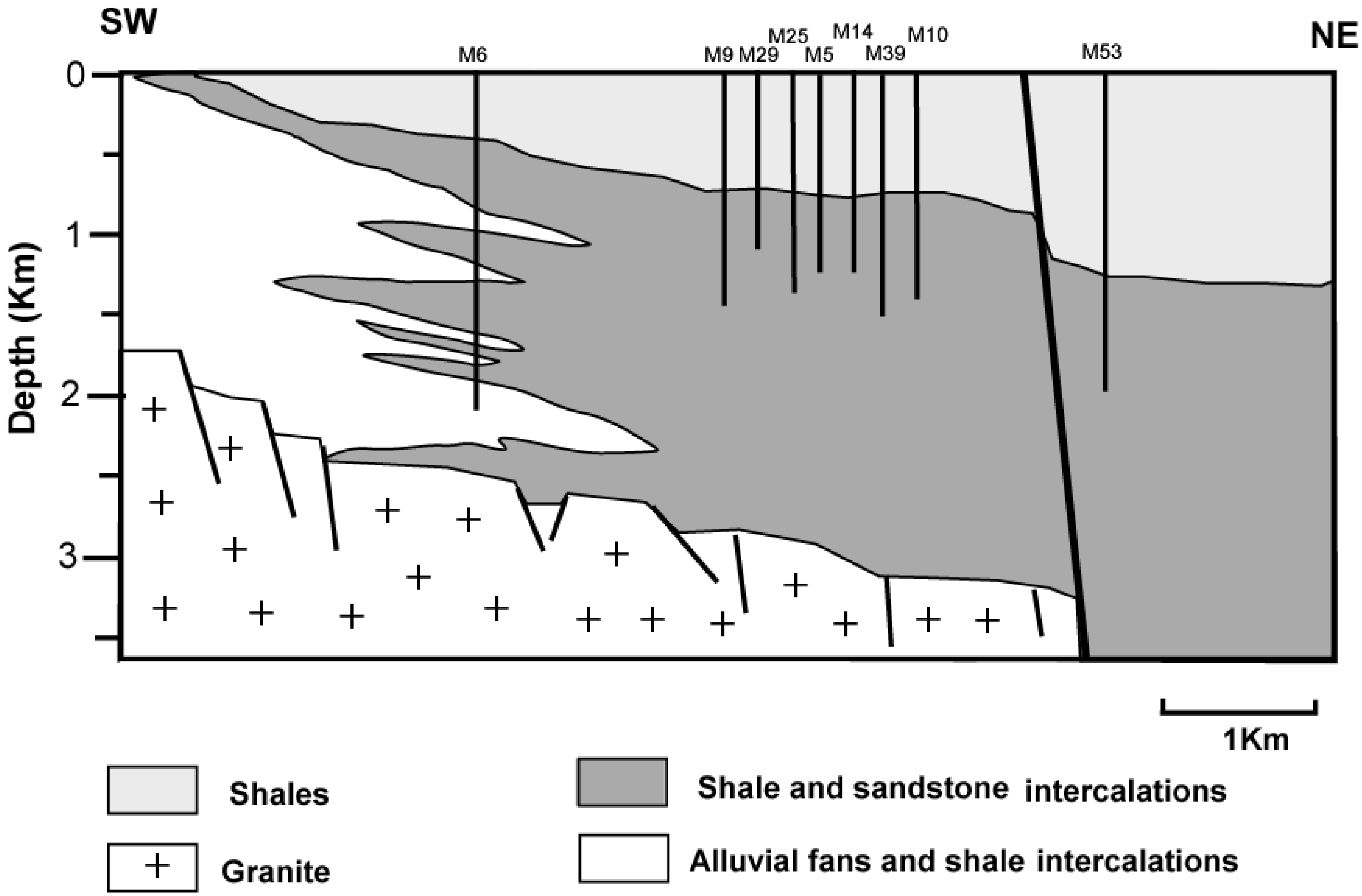
Parametric inversion λ, k, σ **for each unit**

Application to the Cerro Prieto field

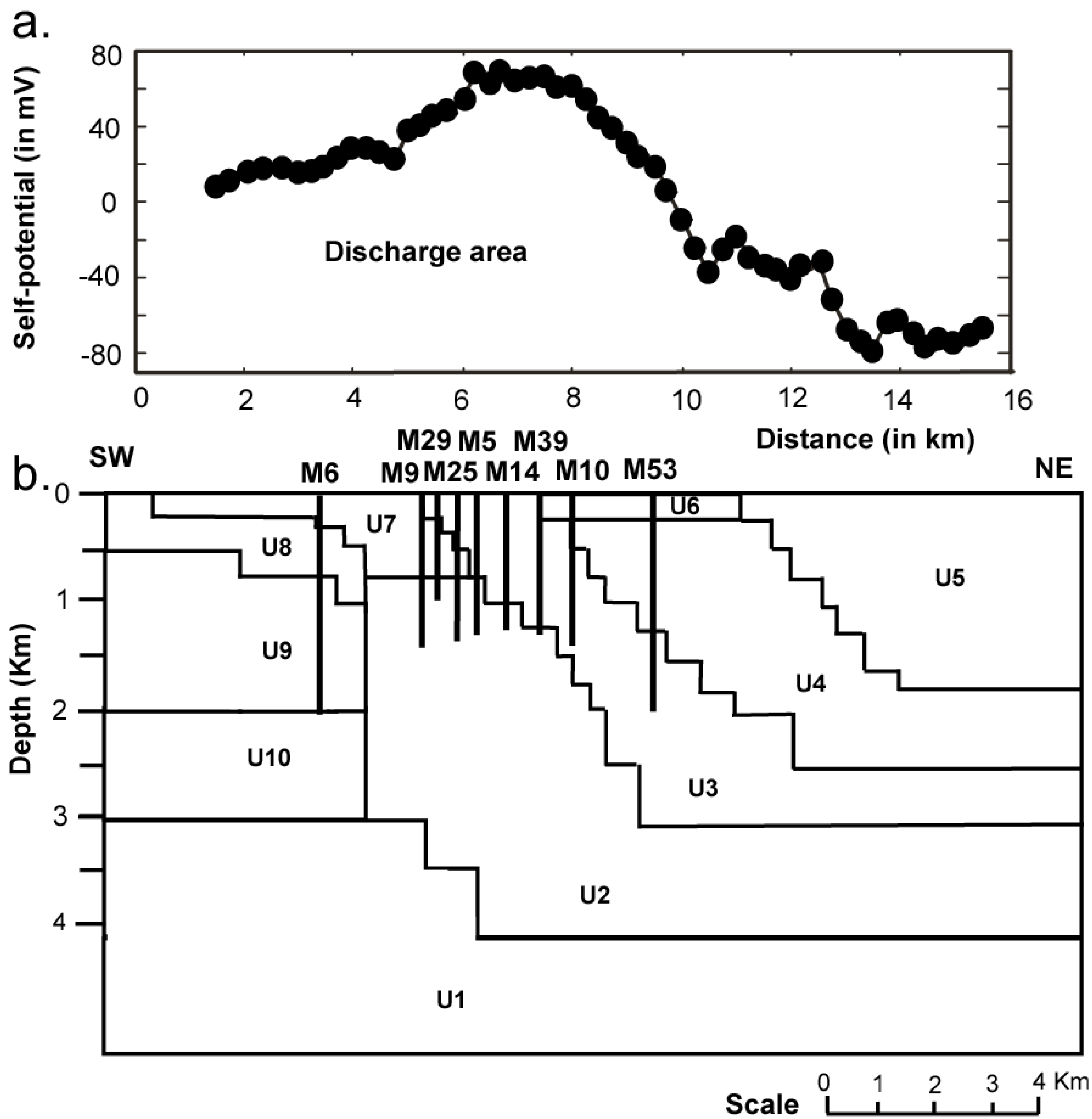




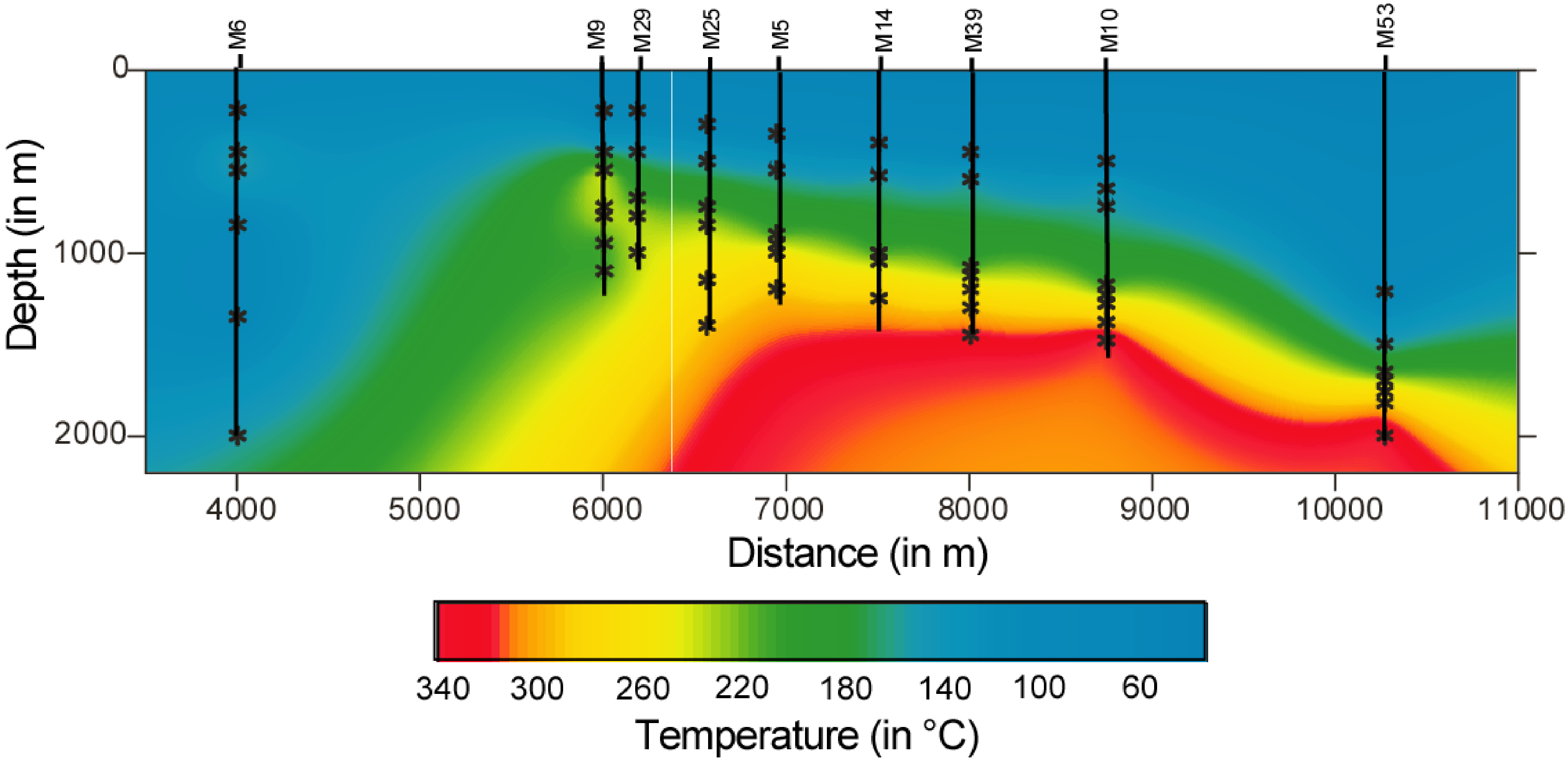
Geological cross-section



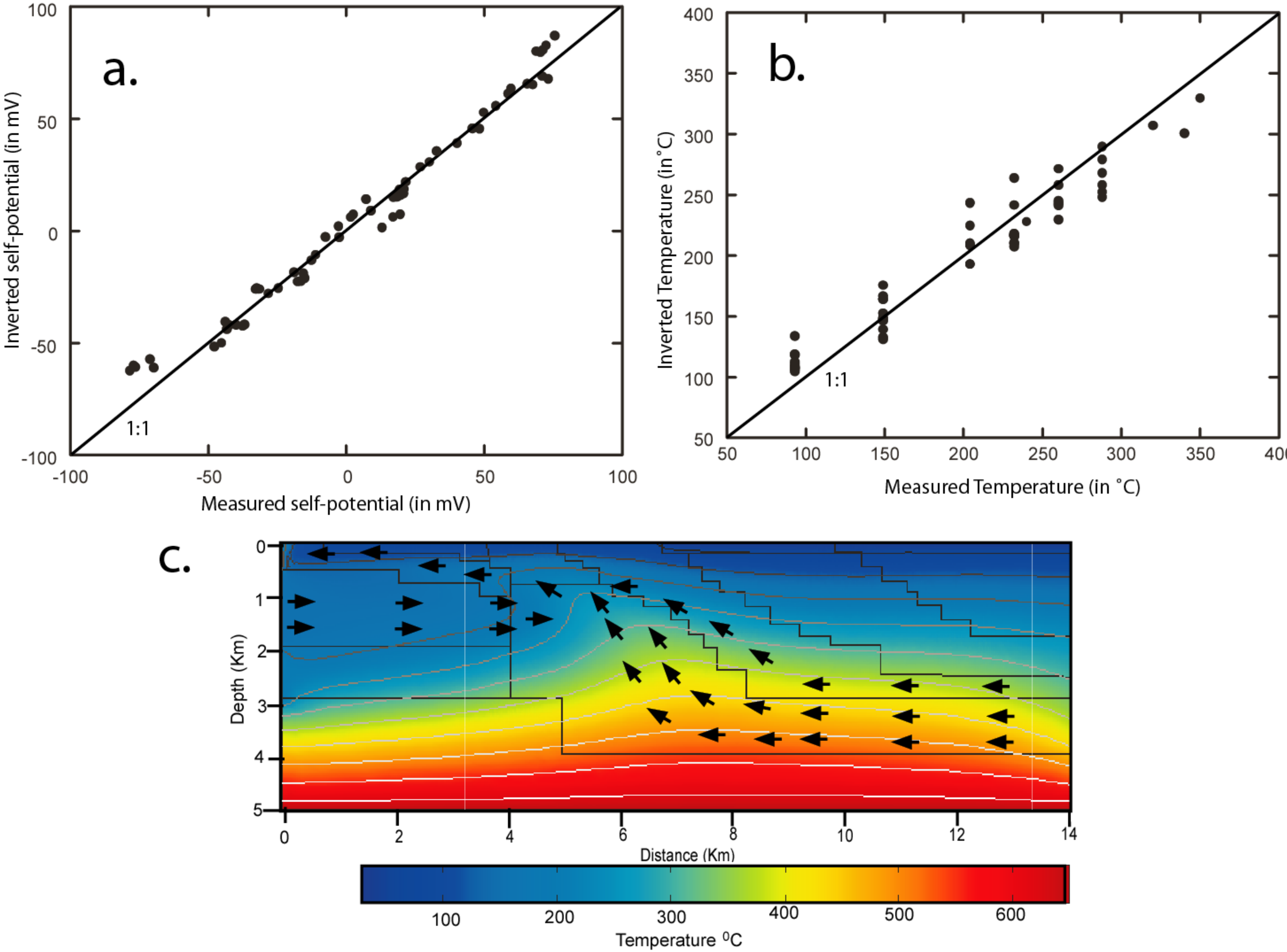
The self-potential data (data from Corwin, USGS, 1976)



The temperature data



Result of the inversion



Modeling seismoelectric waves in a porous material with a Newtonian fluid

Fundamental equations **Time dependence** $\exp(-\mathrm{j}\omega \mathrm{t})$

$-\omega^2 \;\; \rho\mathbf{u} + \rho_f \mathbf{w} \;\; = \nabla \cdot \mathbf{T} + \mathbf{F}$	**Newton's law (1)**
$\mathbf{T} = \;\; \lambda_u \nabla \cdot \mathbf{u} + C\nabla \cdot \mathbf{w} \;\; \mathbf{I} + G\left[\nabla\mathbf{u} + \nabla\mathbf{u}^T\right]$	**Constitutive equation (2)**
$-\omega^2(\rho_f \mathbf{u} + \tilde{\rho}_f \mathbf{w}) - jb\omega\mathbf{w} = -\nabla p + \mathbf{F}_\mathrm{f}$	**Momentum csv equation (3)**
$-p = C\nabla \cdot \mathbf{u} + M\nabla \cdot \mathbf{w} + \mathrm{S}$	**Biot constitutive equation (4)**

Eq. (3) is also Darcy's equation for fluid flow in porous media

$\mathbf{w}$ **Average displacement of the fluid phase**

$\mathbf{u}$ **Average displacement of the solid phase**

Material properties

$$\alpha = 1 - K_{fr} / K_s$$

Biot's coefficient

$$\tilde{\rho}_f = \frac{\rho_f \phi}{a}$$

Apparent density of the pore fluid

$$b = \frac{\eta_f}{k_0}$$

Fluid viscosity to DC permeability ratio

$$K_u = \frac{K_f (K_s - K_{fr}) + \phi K_{fr} (K_s - K_f)}{K_f (1 - \phi - K_{fr} / K_s) + \phi K_s}$$

Undrained bulk modulus

$$C = \frac{K_f (K_s - K_{fr})}{K_f (1 - \phi - K_{fr} / K_s) + \phi K_s}$$

C-Biot's coefficent

$$M = \frac{C}{\alpha} = \frac{K_f K_s}{K_f (1 - \phi - K_{fr} / K_s) + \phi K_s}$$

M-Biot's coefficent

The previous formulation is good
but in 2D it has four unknowns to solve for

w **Average displacement of the fluid phase**

u **Average displacement of the solid phase**

We can look for a formulation with three unknowns

p **The pore fluid pressure**

u **Average displacement of the solid phase**

Reparametrisation in terms of fluid presssure and displacement of the solid phase

$$-\omega^2 \rho_\omega^s \mathbf{u} + \theta_\omega \nabla p = \nabla \cdot \hat{\mathbf{T}} + \mathbf{F}$$

$$\hat{\mathbf{T}} = \lambda(\nabla \cdot \mathbf{u})\mathbf{I} + G\left[\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}}\right]$$

$$\mathbf{T} = \hat{\mathbf{T}} - \alpha p \mathbf{I}$$ **Effective stress tensor**

$$\frac{1}{M}(p+S) + \nabla \cdot \ k_\omega \left[\nabla p - \omega^2 \rho_f \mathbf{u}\right] = \alpha \nabla \cdot \mathbf{u}$$

Material properties

$$k_\omega = \frac{1}{\omega^2 \tilde{\rho}_f + j\omega b}$$

$$\rho_\omega^s = \rho - \omega^2 \rho_f^2 k_\omega$$

$$\lambda = K - \frac{2}{3}G$$

$$\theta_\omega = \alpha - \omega^2 \rho_f k_\omega$$

$\theta_w \nabla p$ **Represents the coupling between the solid and fluid phase**

Description of the seismic source

$$\mathbf{F}(x, y, \omega) = F(\omega)\nabla\ \delta(x - x_0)\delta(y - y_0)$$

$$F(\omega) = \mathrm{FT}\left[(t - t_0)\exp\ -\ \pi f_0 (t - t_0)\ ^2\ \right]$$

Ricker source

$\mathrm{FT}\ \ f(t)$ **Fourier transform of** $f(t)$

t_0 **Time delay of the source**

f_0 **Dominant frequency of the source**

$S = 0$ **No fluid pressure source**
(to avoid EM effects associated with the source)

Electrostatic part

Low frequency EM source (controlled by the frequency of the seismic wave)

The EM disturbances are diffusive

If we are close enought to the sources the EM field is quasistatic

Constitutive equation

$$\mathbf{j} = \sigma \mathbf{E} + \mathbf{j}_S$$

Continuity equation

$$\nabla \cdot \mathbf{j} = 0$$

Field equation: Poisson equation

$$\nabla \cdot (\sigma \nabla \psi) = \nabla \cdot \mathbf{j}_S$$

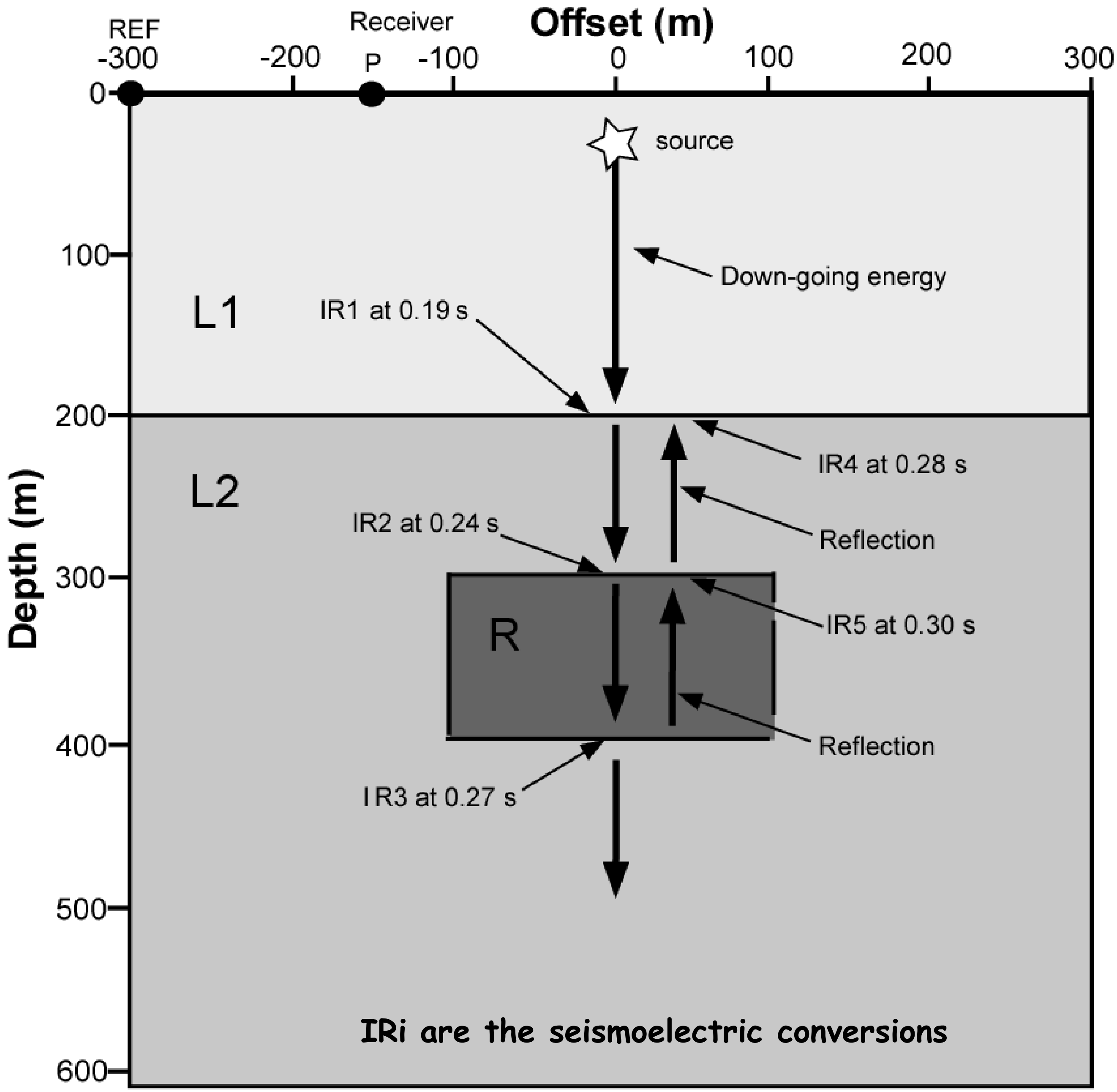
$$\nabla \times \mathbf{E} = 0$$

$$\mathbf{E} = -\nabla \psi$$

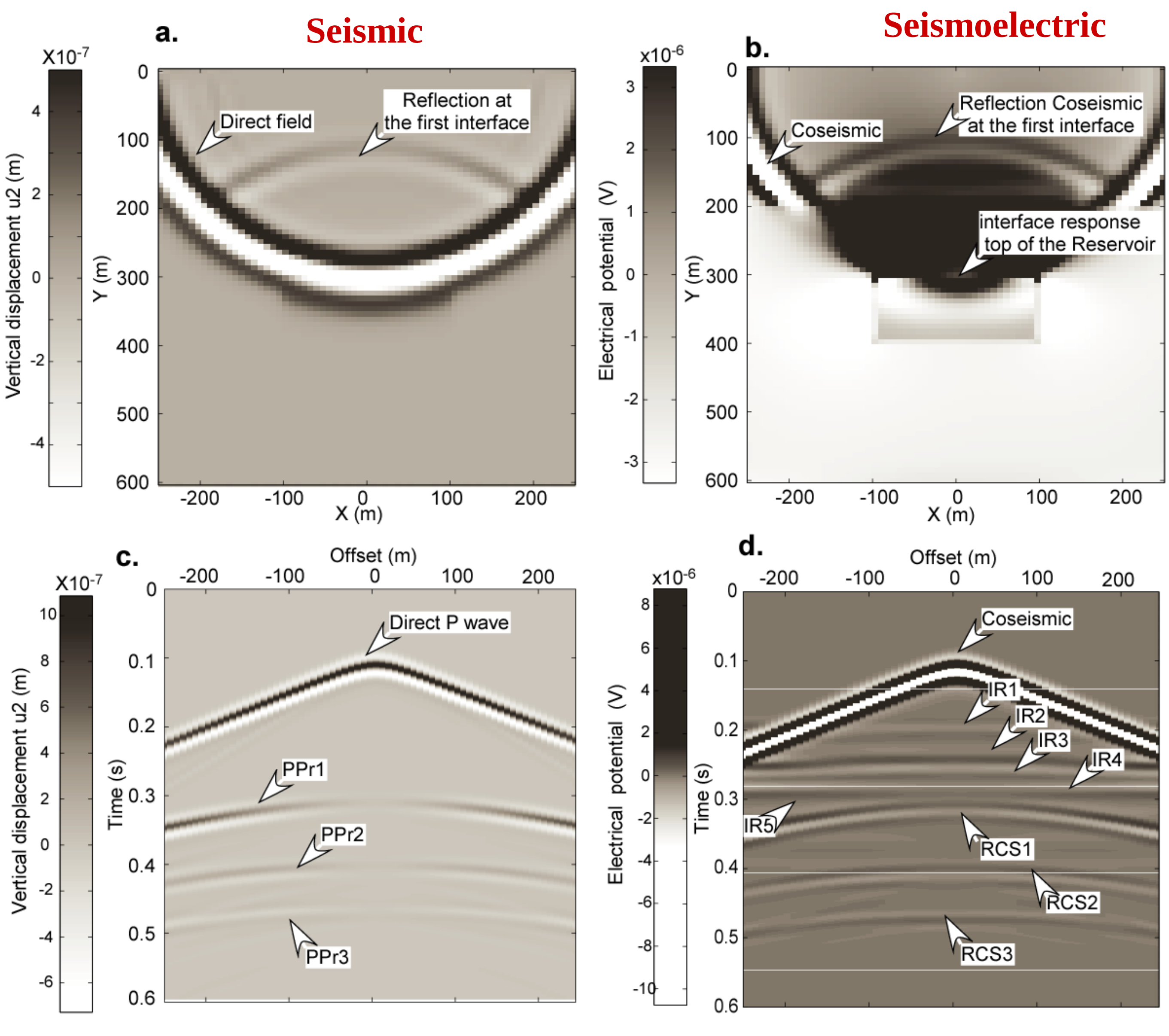
Source current density

$$\mathbf{j}_S = \bar{Q}_V \dot{\mathbf{w}} = -j\omega \bar{Q}_V k_\omega (\nabla p - \omega^2 \rho_f \mathbf{u})$$

Geometry

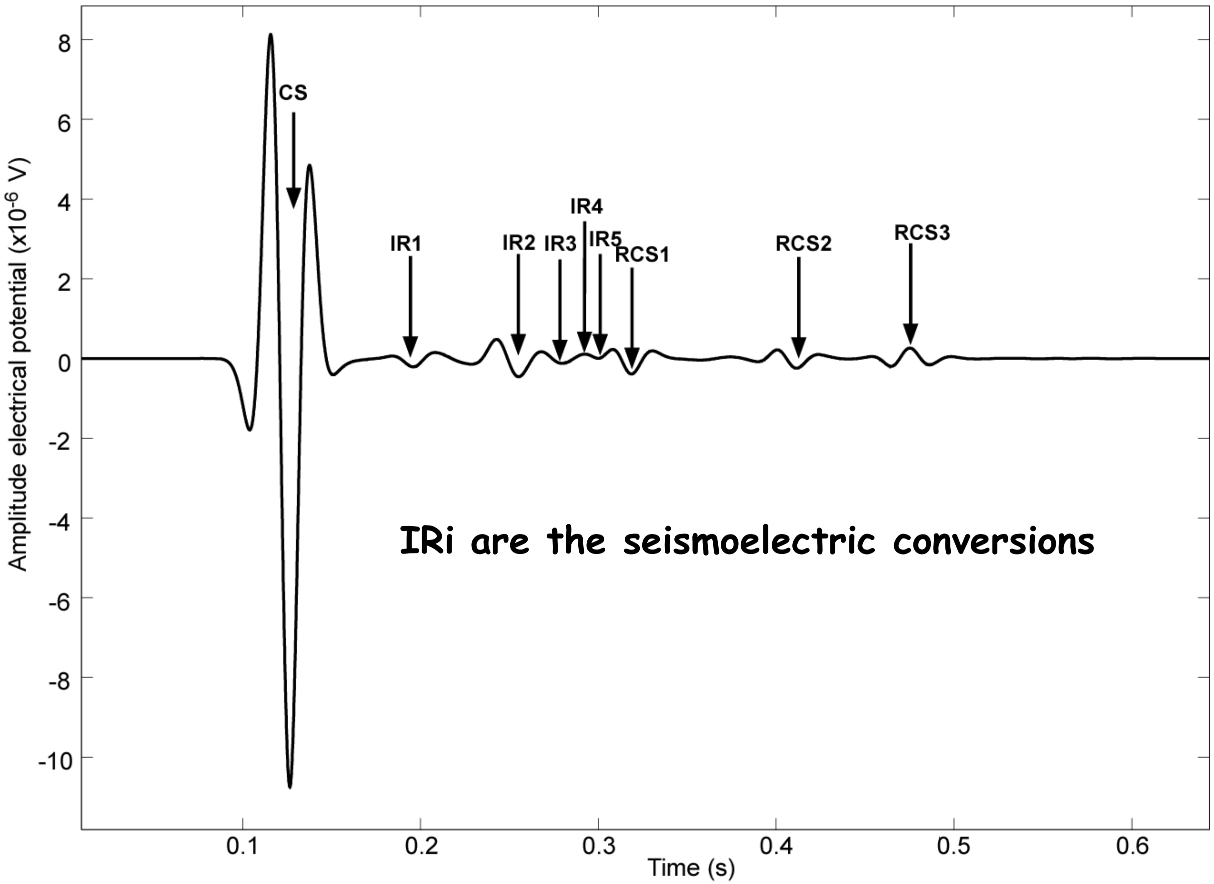

REF
Receiver
P
Offset (m)
-300
-200
-100
0
100
200
300
0
100
200
300
400
500
600
Depth (m)
source
Down-going energy
L1
IR1 at 0.19 s
L2
IR4 at 0.28 s
Reflection
IR2 at 0.24 s
R
IR5 at 0.30 s
Reflection
I R3 at 0.27 s
IRi are the seismoelectric conversions

Example of numerical modeling



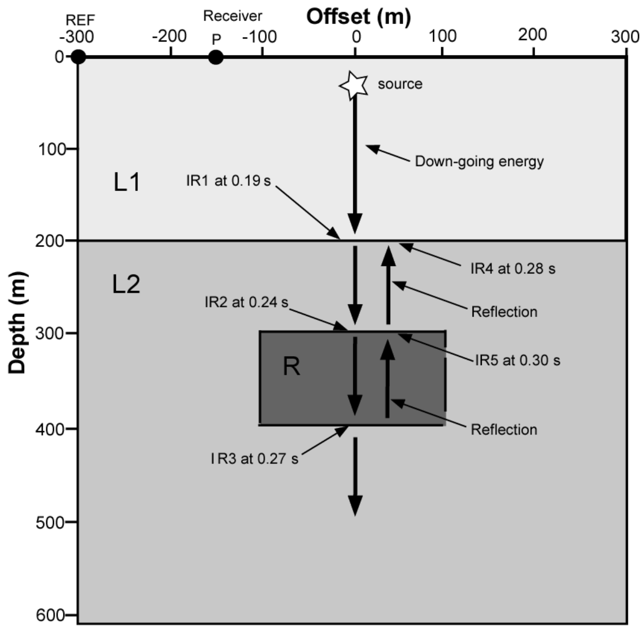
IRi are the seismoelectric conversions

Typical electrogram at observation point P



IRi are the seismoelectric conversions

Receiver P



Geometry

Stochastic inverse modeling (1/2)

Use of a Bayesian framework (Tarantola, 2005)

Joint inversion of seismic and seismoelectric signals

The boundaries are assumed to be determined prior the inversion

Parametric inversion of the material properties for each unit

Vector of model parameters

$$\mathbf{m} = \{\log k_0; \log \mathrm{it}\,\phi; \log \sigma; \log K_{fr}; \log K_f; \log K_S; \log G\}$$

$$\log \mathrm{it}\,\phi = \log\left(\frac{\phi}{1-\phi}\right)$$

Permeability, porosity, conductivity, 3 bulk moduli, shear modulus

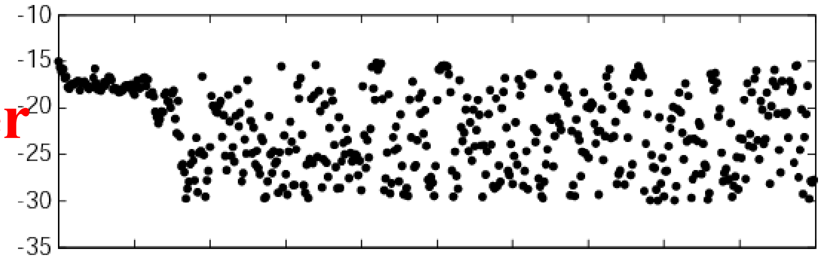
Stochastic inverse modeling (2/2)

Use of MCMC approach (Adaptative Metropolis Algorithm)

25000 realizations

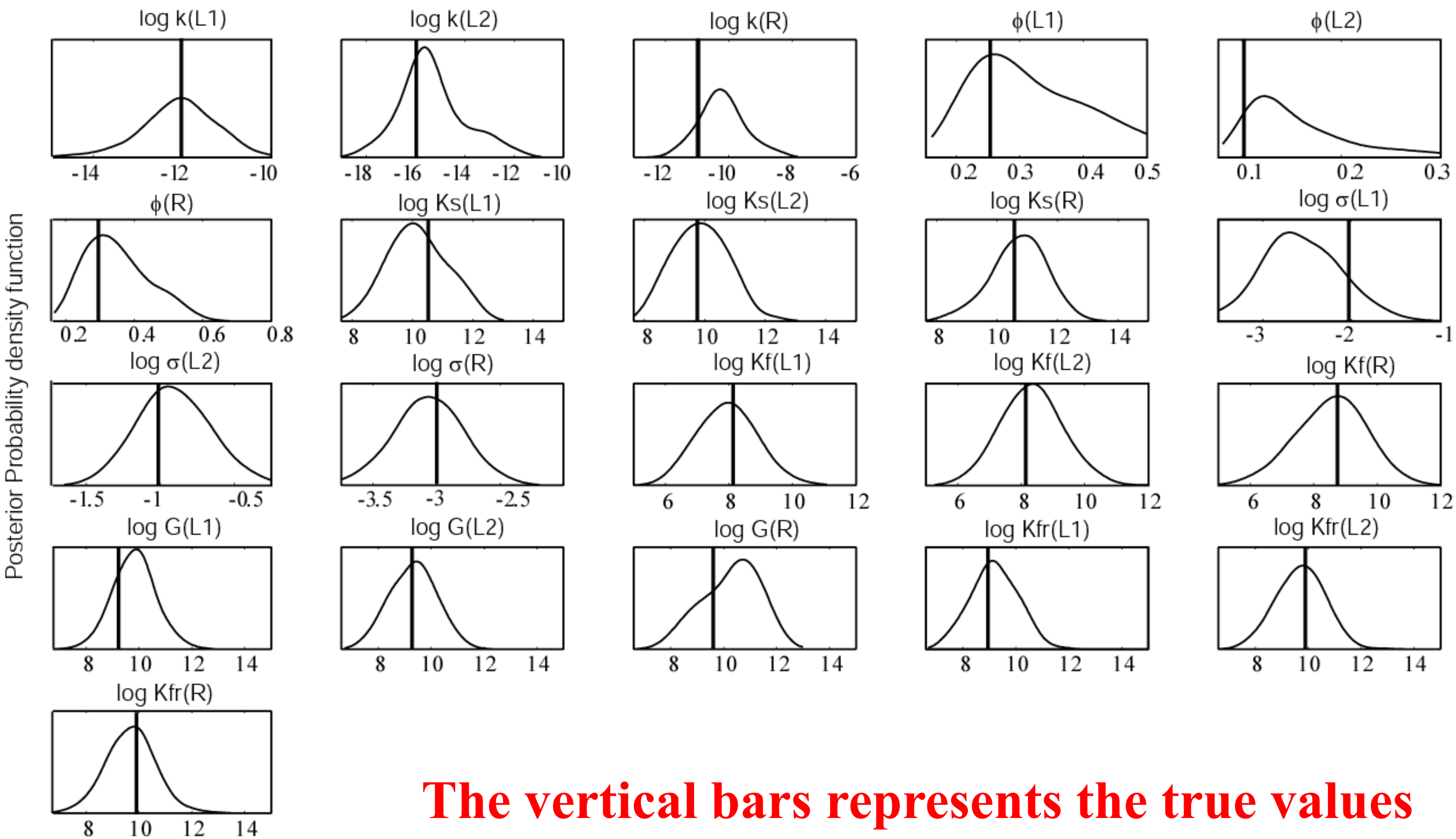
21 unknowns

Model parameter



Realizations

Stochastic full wave form inversion to retrieve both the permeability and mechanical properties



The vertical bars represents the true values