Simulation du système cardiovasculaire et modélisation réduite

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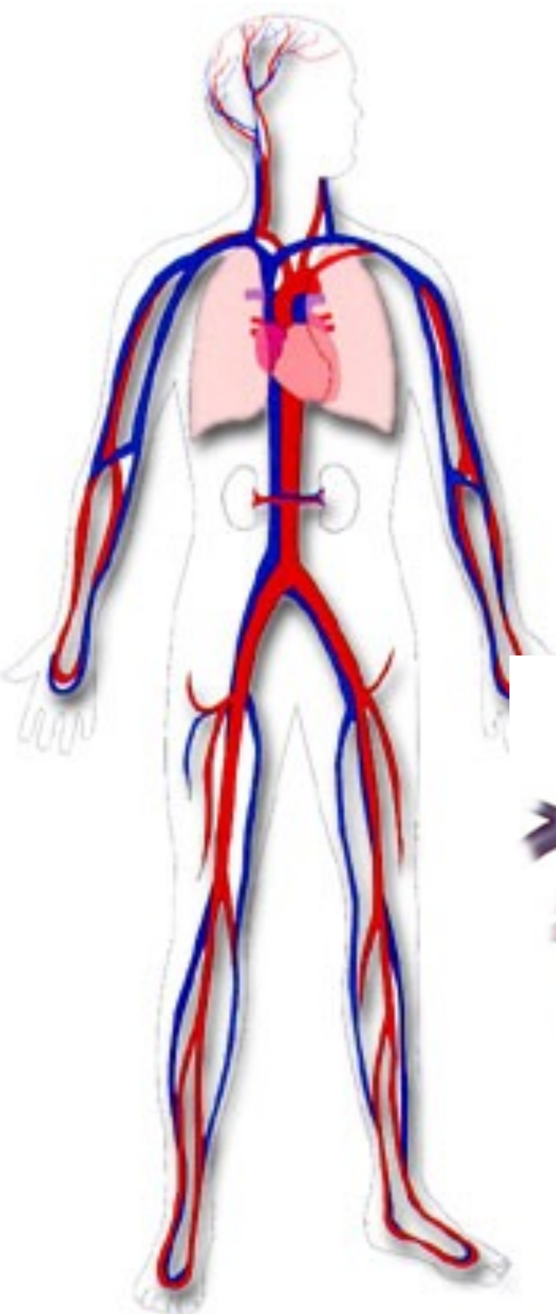
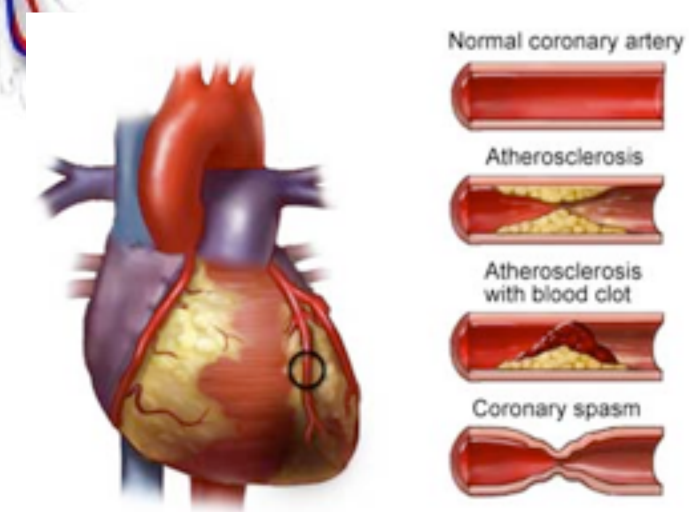
France

Cardiovascular system

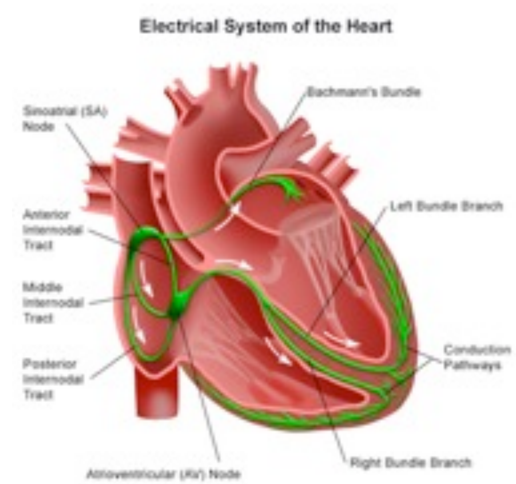


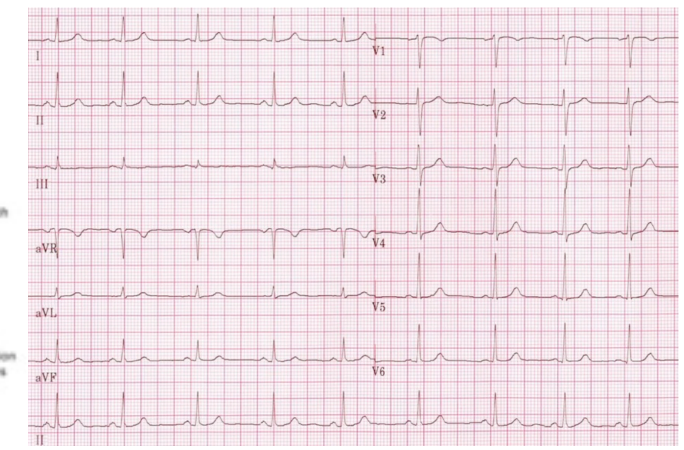
Aneurisms



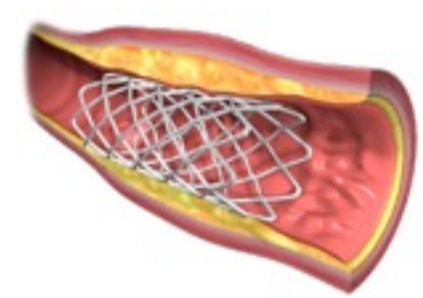
Atherosclerosis

Vascular diseases





Electrophysiology
(Arythmia, drug effect,...)



Biomedical devices

Outline

- Computational Hemodynamics
  - Motivations
  - Inverse problems in fluid-structure
- Electrocardiograms (ECGs)
- Reduced order modeling
  - Motivations
  - The Approximated Lax Pair method

Outline

- **Computational Hemodynamics**
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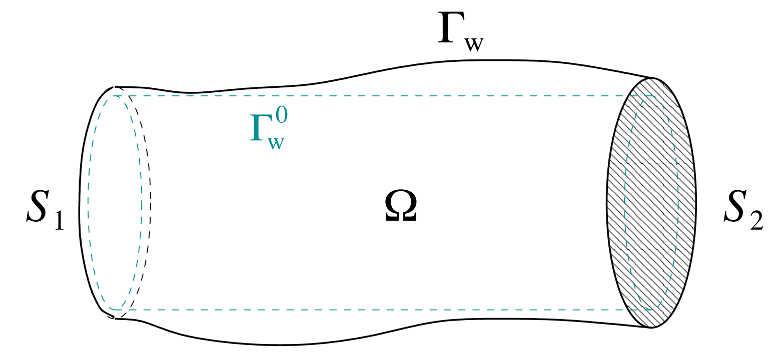
Computational hemodynamics:
Hierarchy of models

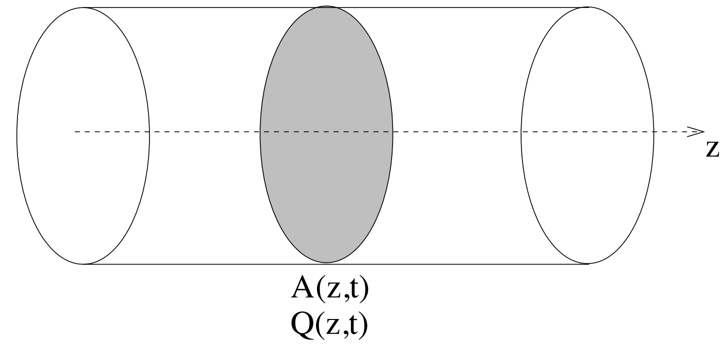
- **3D Model** (Navier-Stokes + Nonlinear elasticity)

  Variables : velocity, pressure, displacement



- **1D Model** (Euler equation, 1775)

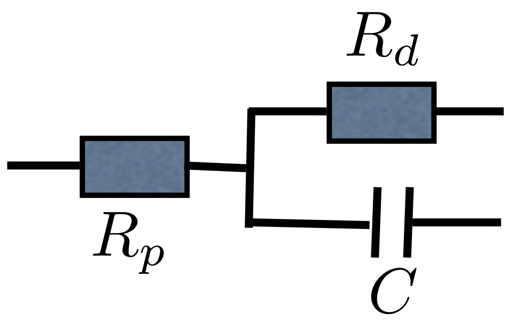
  Variables : flow rate, mean pressure, section area



- **0D Model** (Ordinary Differential Equations)

  Variables : flow rate, pressure drop

  (compartment modeling or **boundary conditions**)



Three-dimensional models

Before 2000 :

- **Pioneer works:** Navier-Stokes equations, simple boundary conditions, rarely on real anatomies

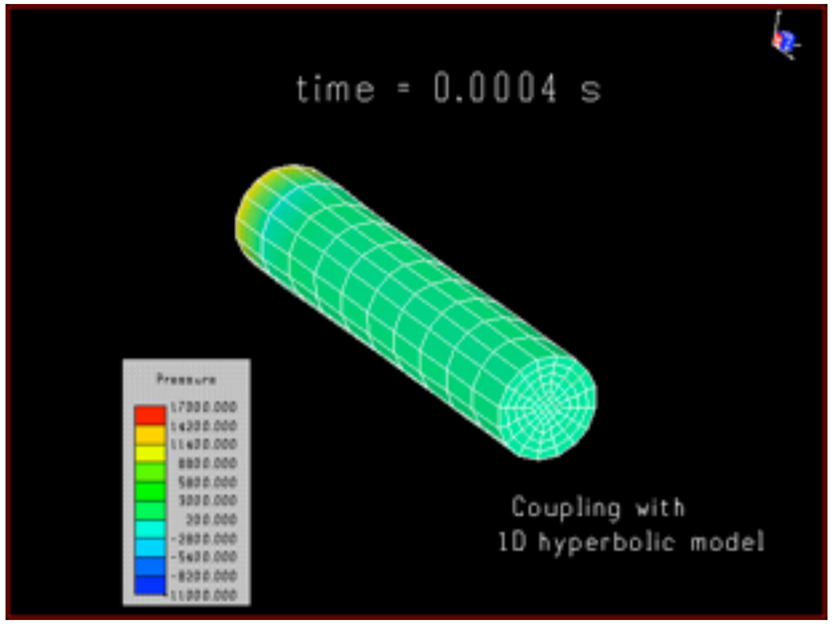
2000 - 2010 :

- **Patient-specific anatomies:** homemade solutions; free (VMTK) or commercial software (Amira, Mimics,...)
- **Fluid-solid interaction:** tremendous progress, now present in commercial software (not very efficient though!)
- **Boundary conditions:** many workarounds... still an open topic!
- **Main applications:** stents, aneurysms, ...

Today (and tomorrow) :

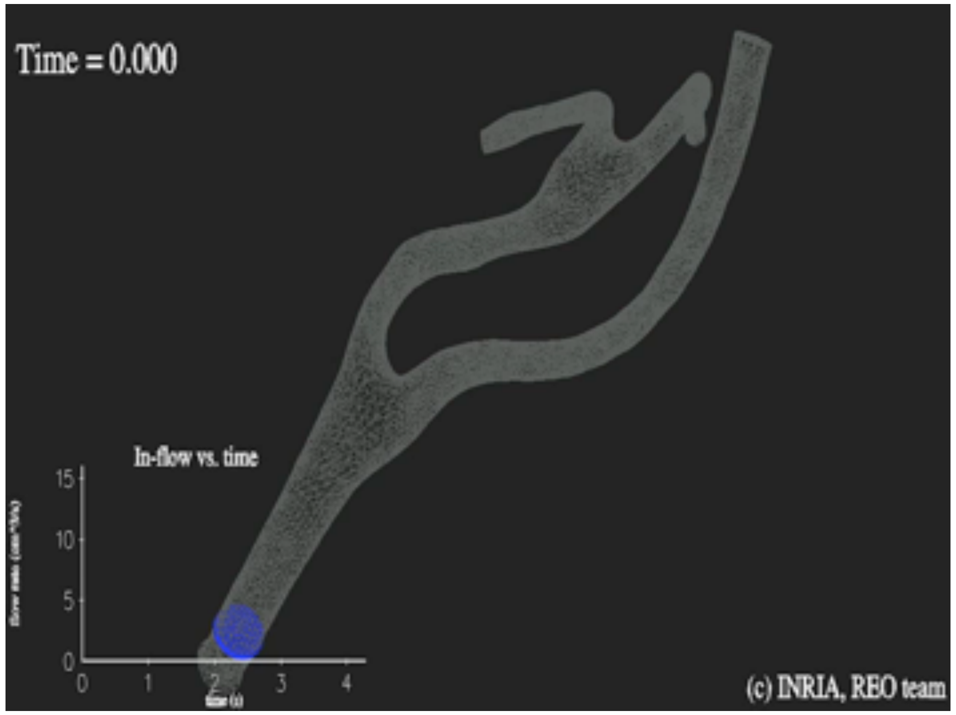
- **Diagnosis**: assimilate medical data in simulations
- **Prognosis & planning**: long term evolution, adaption, remodeling,...
- Find problems where all these works can be *really* useful!

Progress in FSI



1999

2003

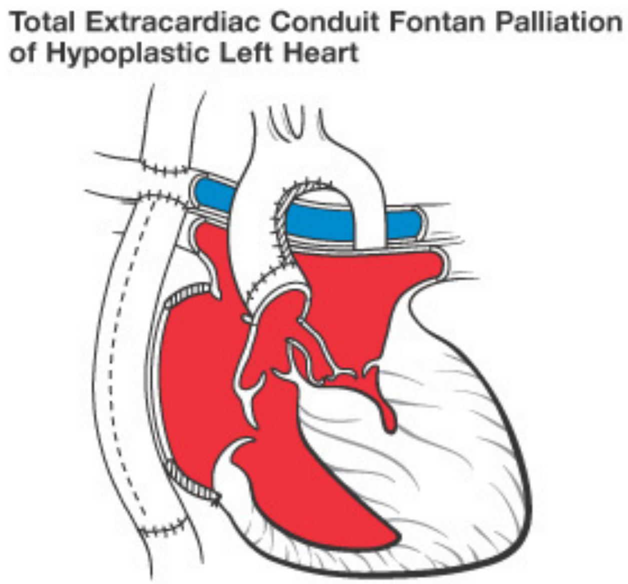


2007

	2001	2003	2007	2012
Computational time	100	20	4	1

Only due to progress in algorithms !

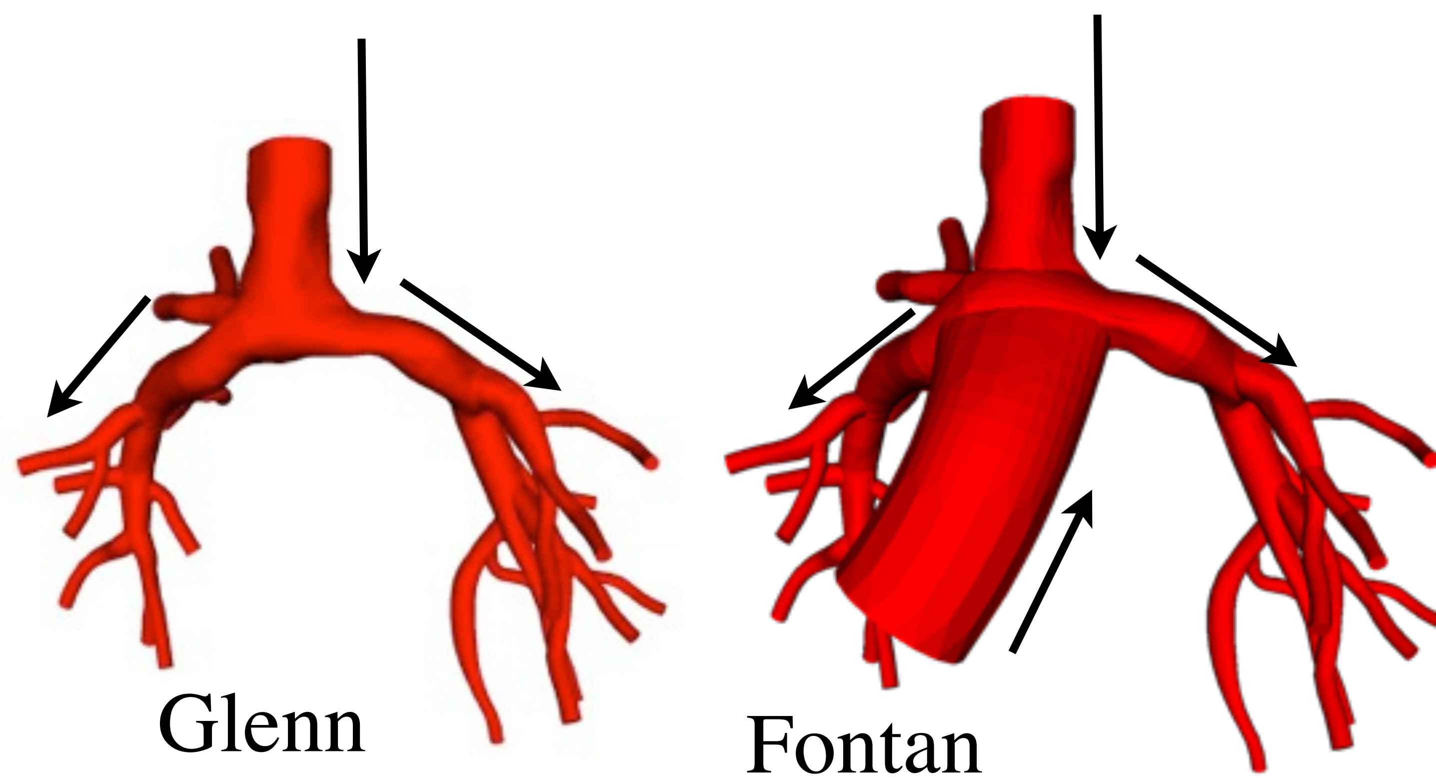
1st example: surgical planning



http://www.americanheart.org

Glenn-Fontan surgery

- congenital heart disease
- multi-step complex procedure



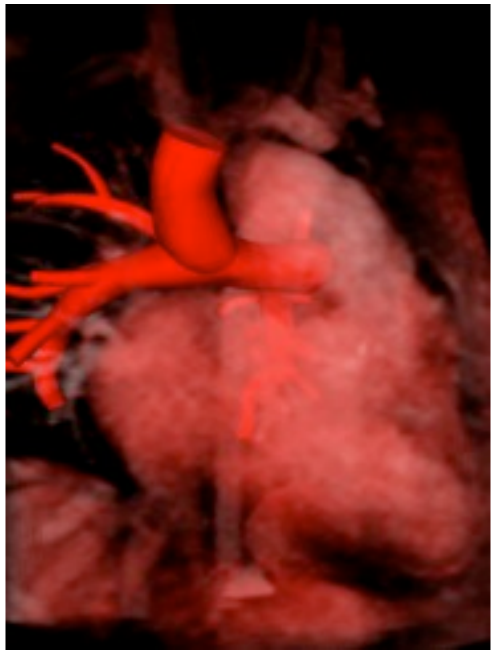
Troianowski, Taylor, Feinstein, Vignon-Clementel
Stanford/INRIA

Numerical simulations

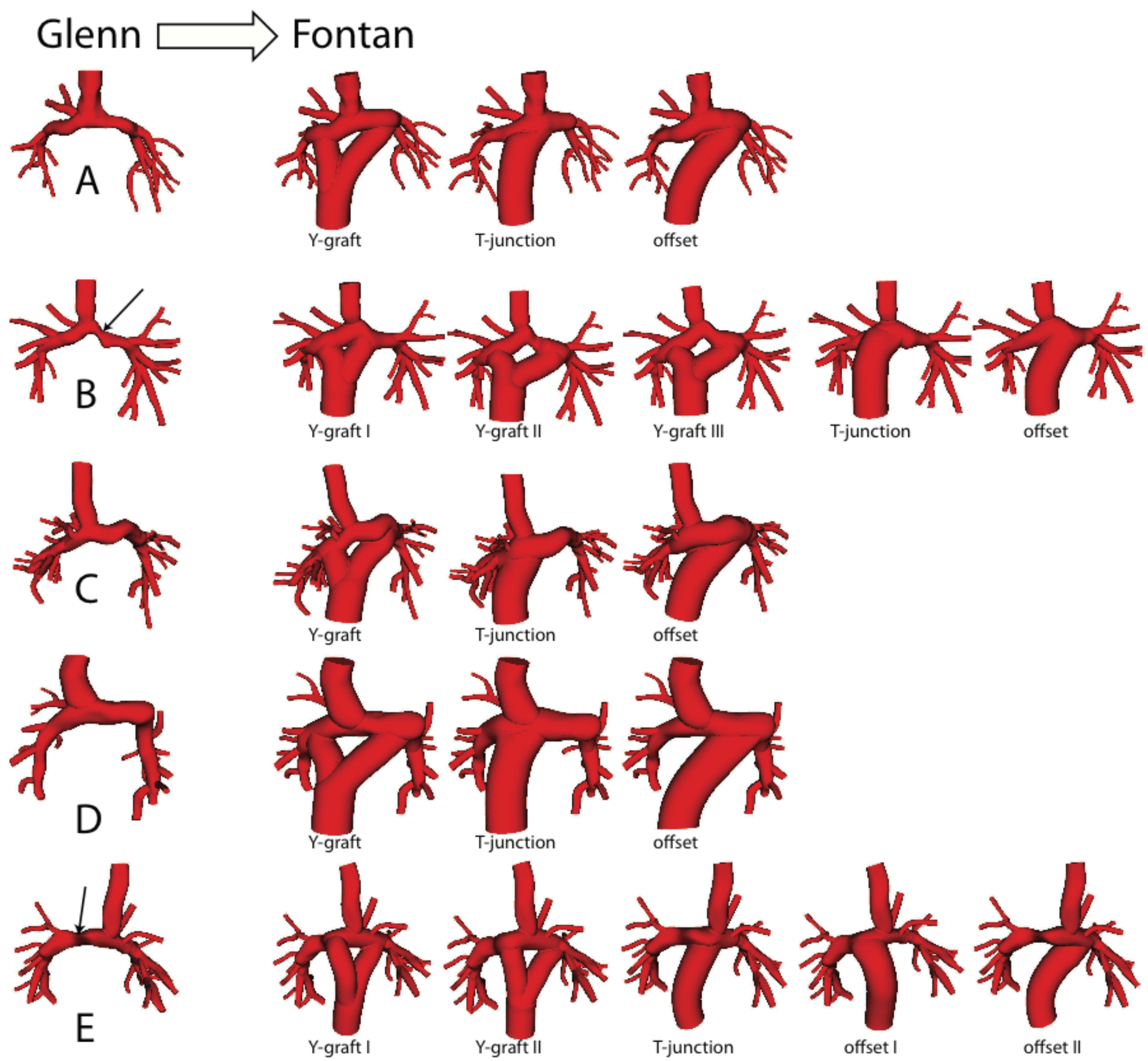
- Patient-specific geometries
- Forecast pressure drop, flow split, wall-shear stress

Geometry from 5 patients MRI data

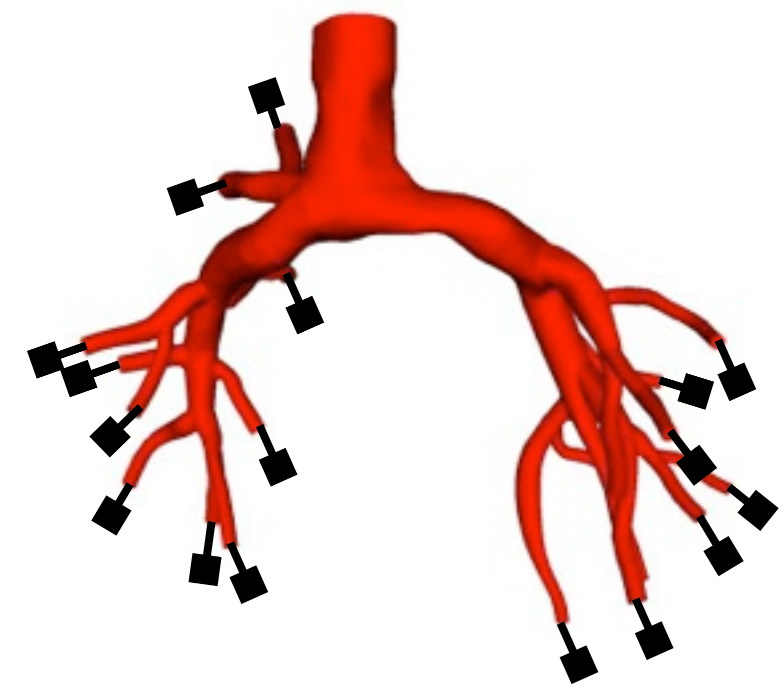


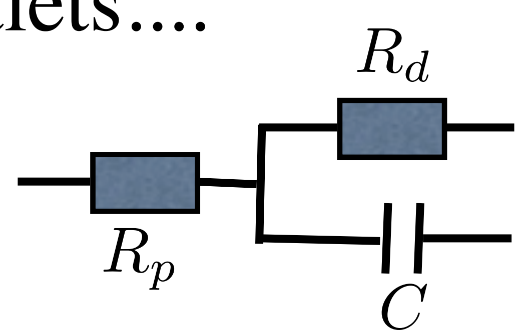

I. Vignon-Clementel, A. Birolleau,
G. Troianowski (INRIA & Stanford)



Challenge:



- About 20 “outlets”
- 3 parameters per outlets....

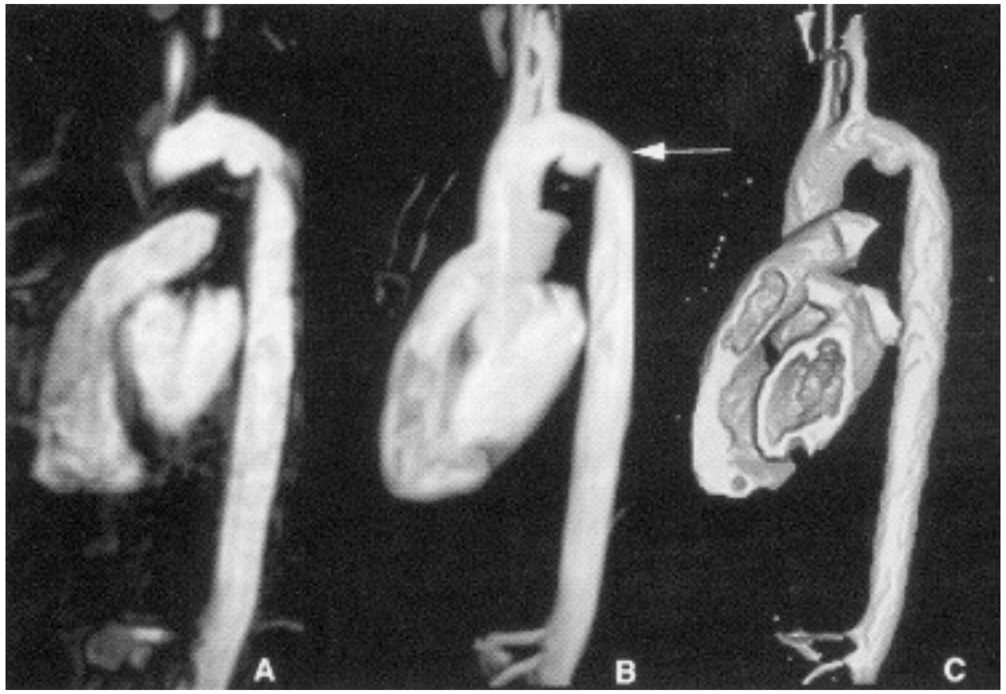


Y-graft in general improves:

- Energy efficiency
- Flow distribution
- SVC pressure under rest & exercise

Troianowski, Taylor, Feinstein, Vignon-Clementel

2d example: avoid clinical exams ?

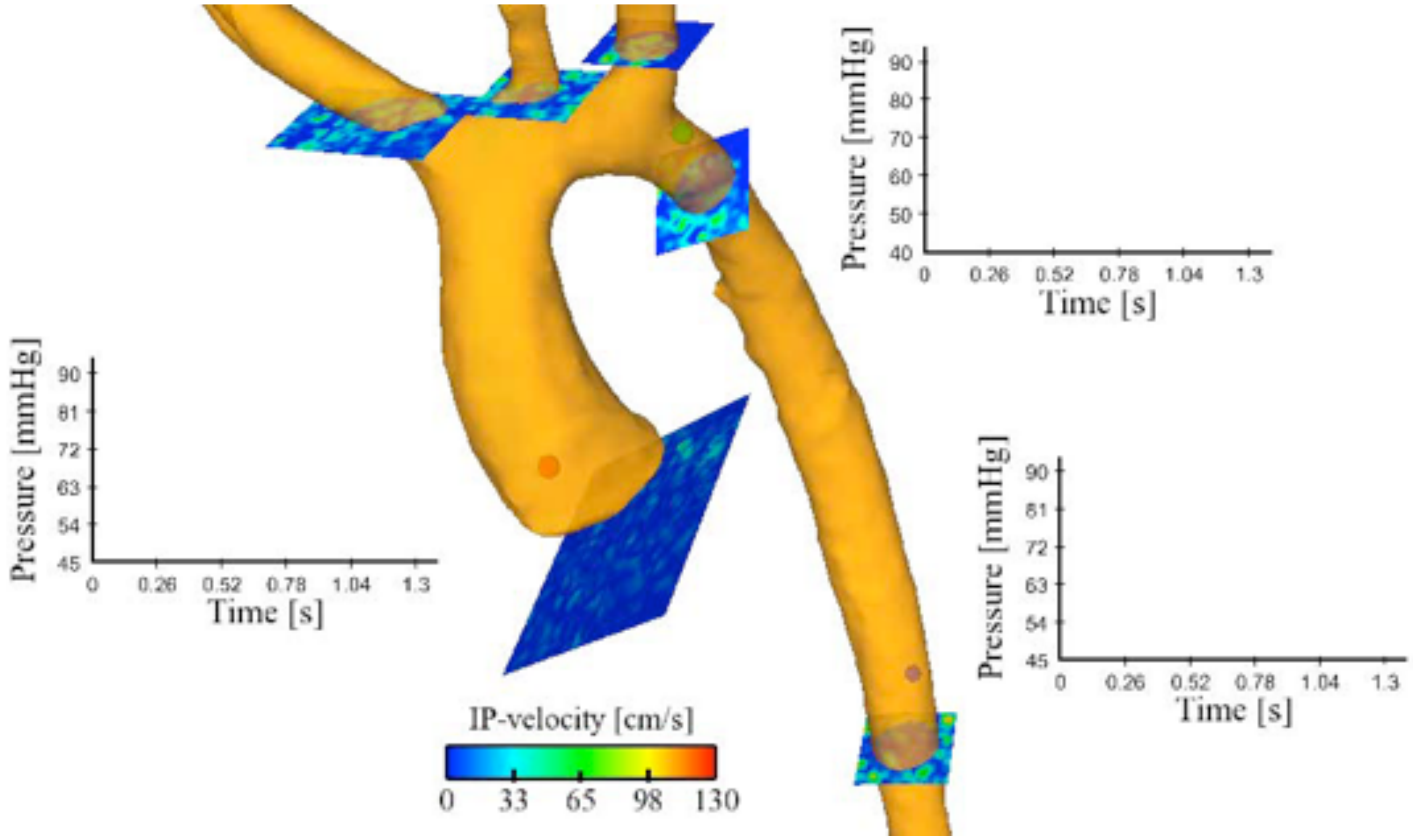


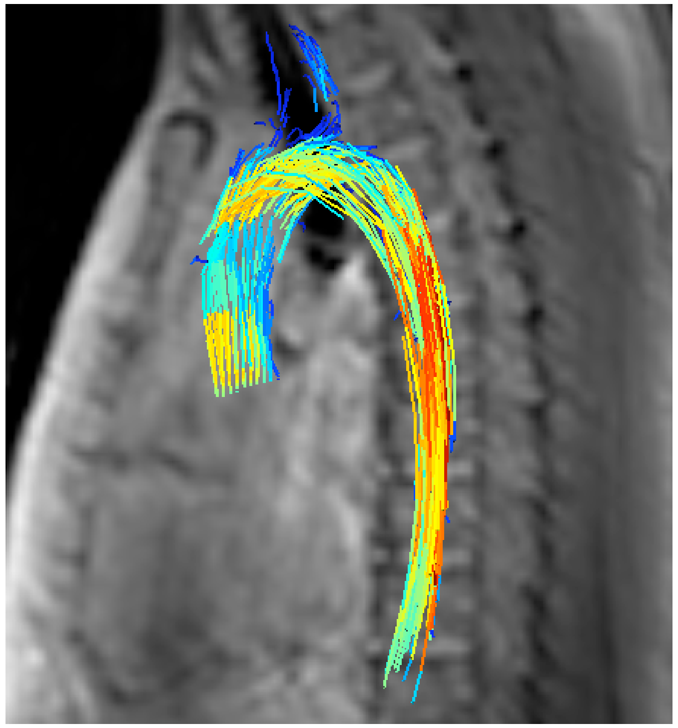
Source: O. Peruta

- **Example:** aortic coarctation
- After surgical repair, patients must be followed on a regular basis
- Exercise test is often necessary to assess the patient condition
- **Question:** With computer simulations, can we extrapolate the rest test to avoid the stress test ?
- Maybe... if we are able to evaluate the artery wall stiffness

Collaboration with R. Hose, I. Valverde, P. Beerbaum (euHeart project)

Aortic coarctation





Data: I. Valverde, P. Beerbaum (KCL).
Segmentation and processing: R. Hose et al. (Sheffield), C. Bertoglio (INRIA) (euHeart project)

Medical data assimilation

Measurements

- medical imaging (CT, MRI, ...)
- blood flow (US, PC-MRI,...)
- pressure (catheter)
- ...

Models

- Navier-Stokes equations
- Solid mechanics
- ...

Estimate parameters

- artery wall stiffness
- boundary condition
- ...

Access to hidden quantities

- pressure
- wall stress

Improve measures

- regularization
- interpolation
- ...

Data assimilation

- FSI dynamical system: $\left\{ \begin{array}{rcl} B\dot{X} & = & A(X,\theta) + R \\ X(0) & = & X_0 \end{array} \right.$
- Time discretization: $X^{n+1} = F^{n+1}(X^n, \theta)$
- **State variable**: X=$[\boldsymbol{u}, p, \boldsymbol{d}^f, \boldsymbol{d}, \boldsymbol{v}]$
- **Parameters**: θ = [Young modulus, viscosity, boundary conditions, $\ldots$]
- **Uncertainties** on the initial condition X_0 and the parameters θ
- Partial **observations** of X: $\quad Z = H(X)$

Data assimilation

- Uncertainties: $\zeta = [\zeta_X, \zeta_\theta]$

$$\begin{cases} X_0 &= \hat{X}_0 + \zeta_X \\ \theta &= \hat{\theta} + \zeta_\theta \end{cases}$$

- Minimize

$$J(\zeta) = \frac{1}{2}\int_0^T \|Z - H(\hat{X})\|_W^2 \, dt + \frac{1}{2}\|\zeta\|_P^2$$

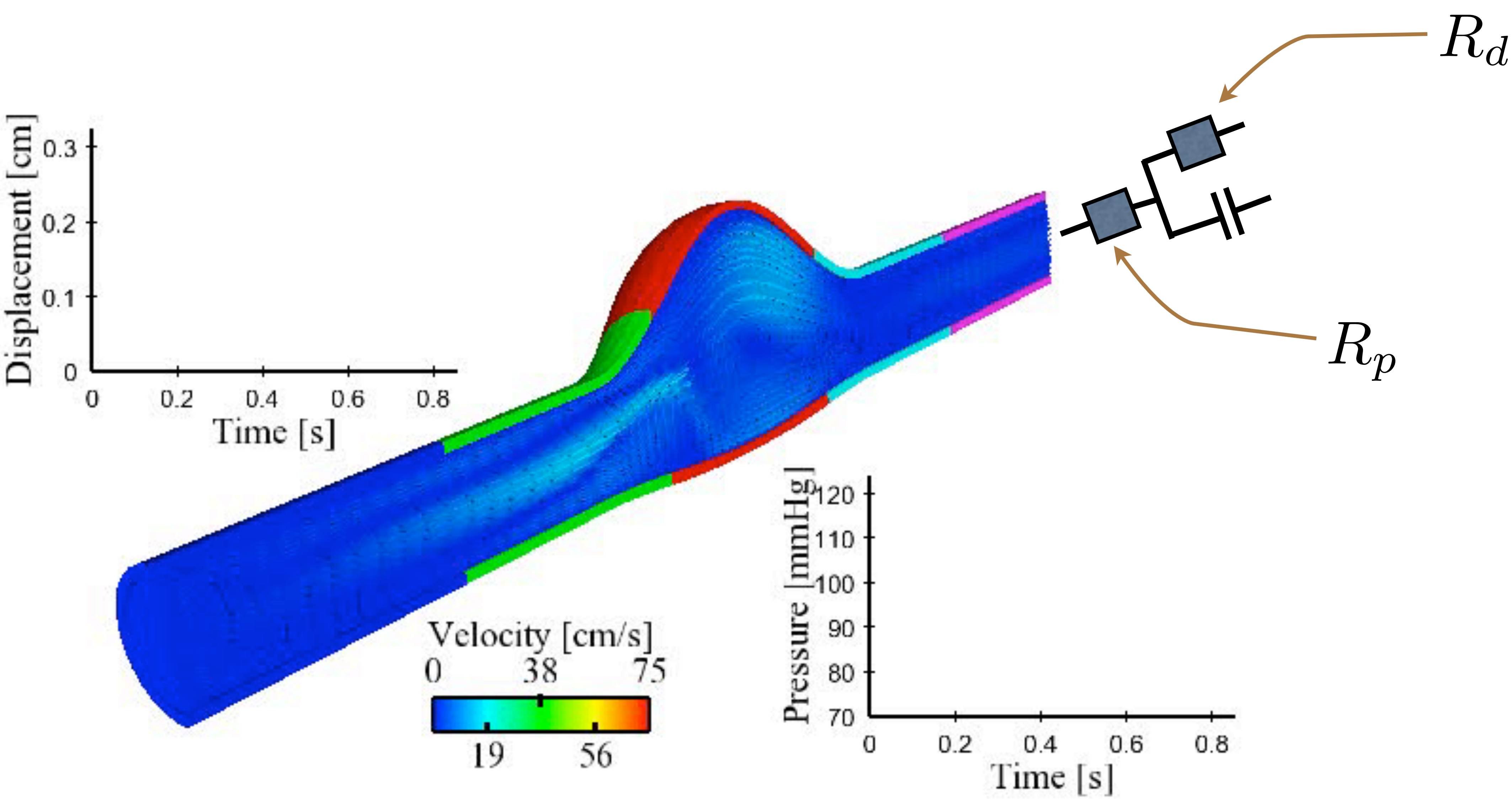
where $\hat{X} = \hat{X}(\zeta)$ is solution to the problem.

- **Variational approach:**
  - Optimization algorithms
  - Usually based on gradient (adjoint equations)
- **Filtering approach:**
  - Sequential correction of the state and the parameters

Example: Compliance estimation

Parameter estimation

- Parameter estimation: Young modulus $E_i = 2^{\theta_i}$ in 5 regions
- Observations: wall displacement on fluid-solid interface

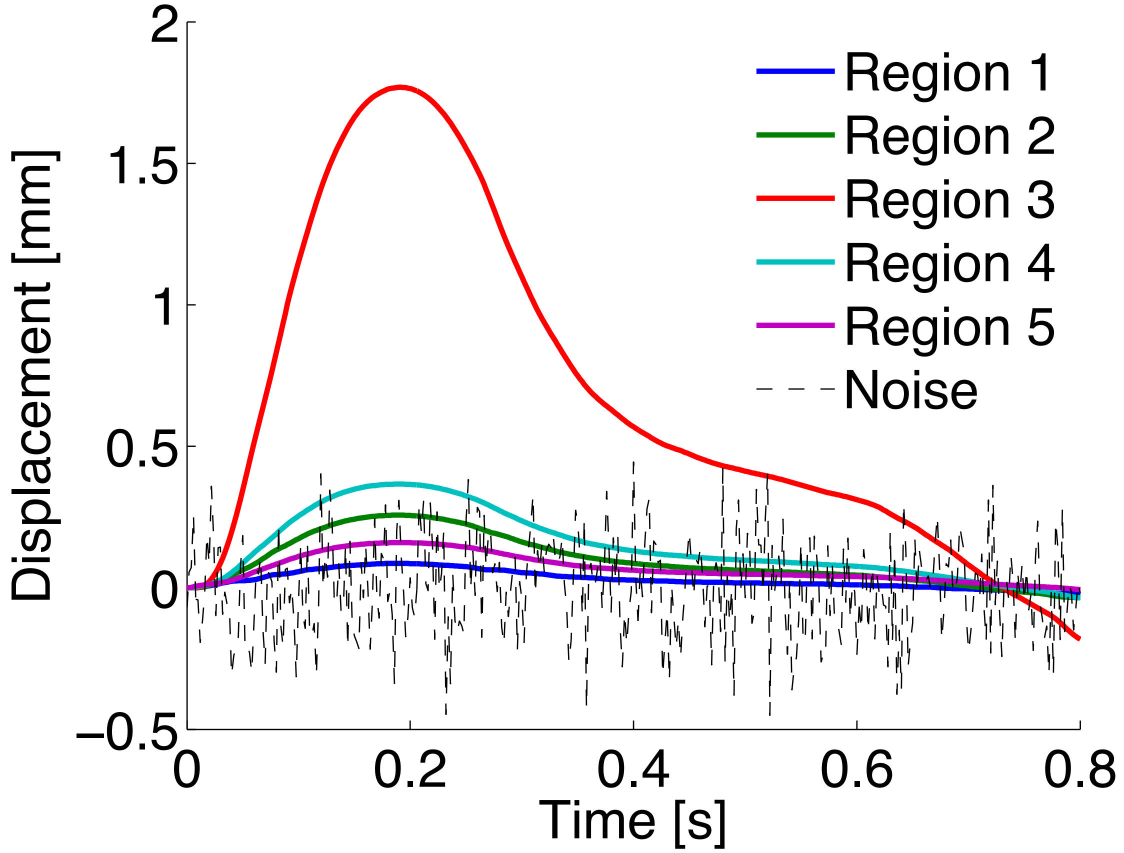


Simulation : C.Bertoglio

Example: Compliance estimation

Parameter estimation

- Synthetic data
- Gaussian noise (10%) and time resampling $\delta t_{obs} = 10\delta t$

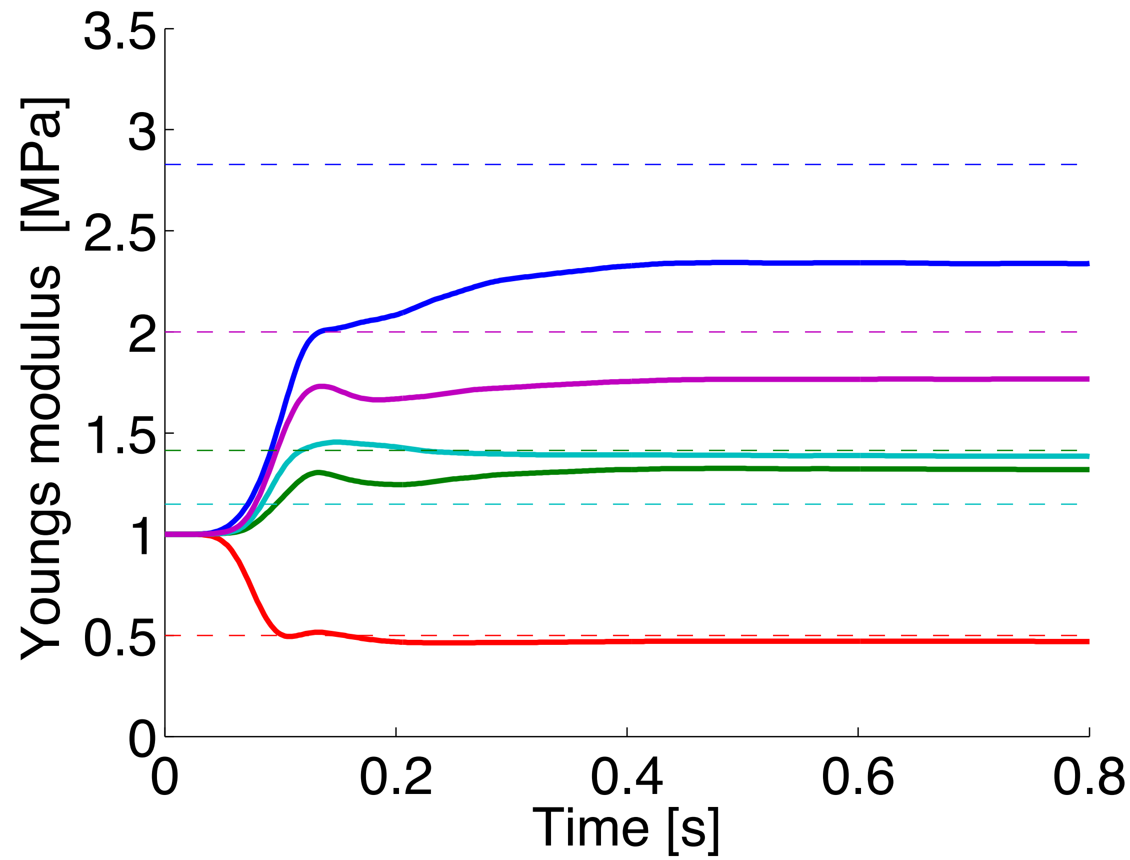


Region	SNR
1	0.43
2	1.25
3	8.8
4	1.8
5	0.75

- Two tuning coeffecients:
  - A priori covariance on parameters: $\alpha\mathbb{I}$
  - Gain: β

Example: Compliance estimation

Parameter estimation



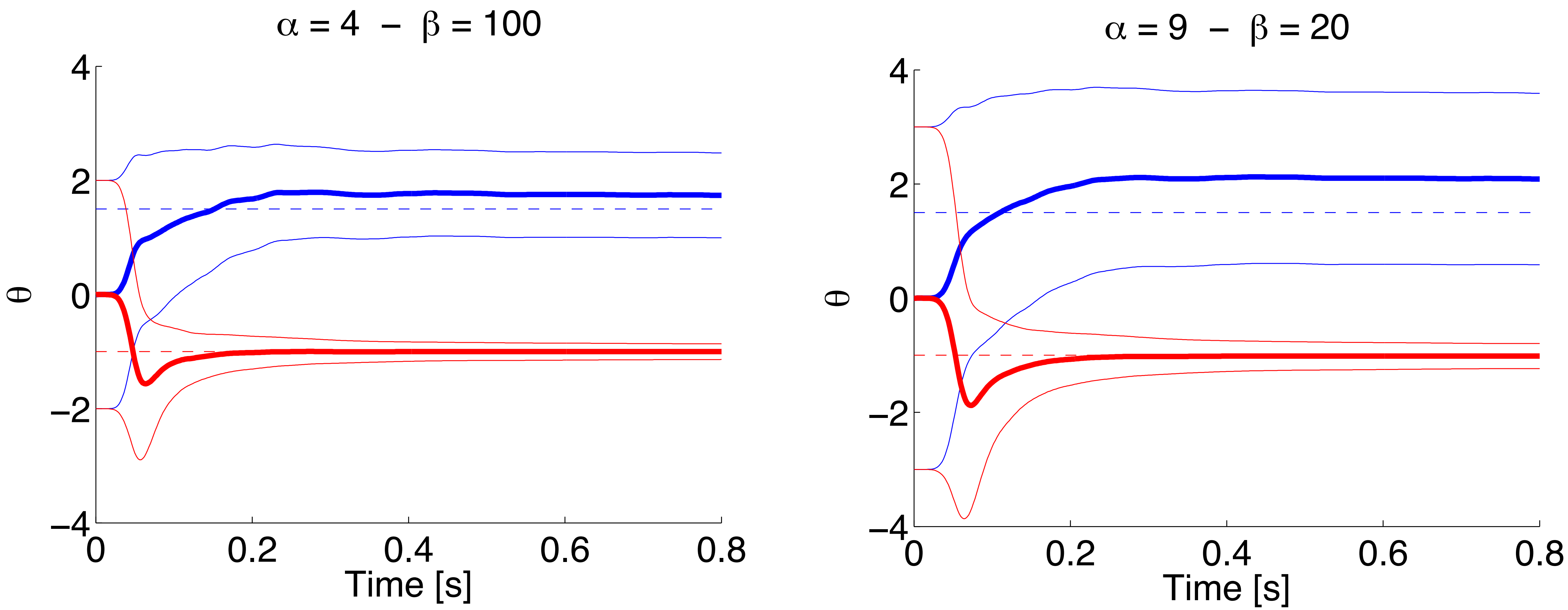
$$\alpha = 9, \beta = 1$$

Simulation : C.Bertoglio

Example: Compliance estimation

Parameter estimation

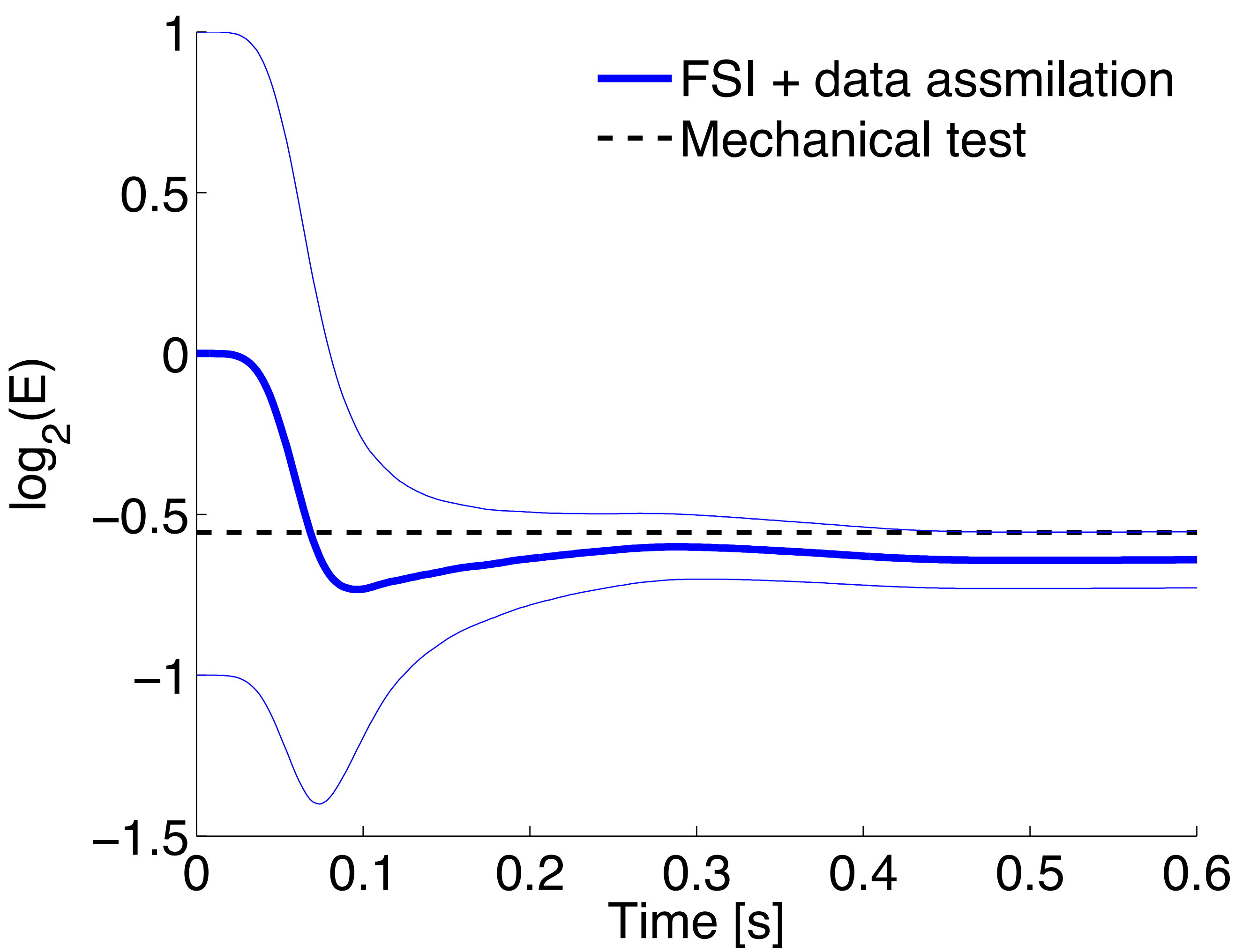
- The filtering algorithm provides an *a posteriori covariance* P_n^θ
- We can plot $\hat{\theta}_n \pm \sqrt{\mathrm{diag}(P_n^\theta)}$

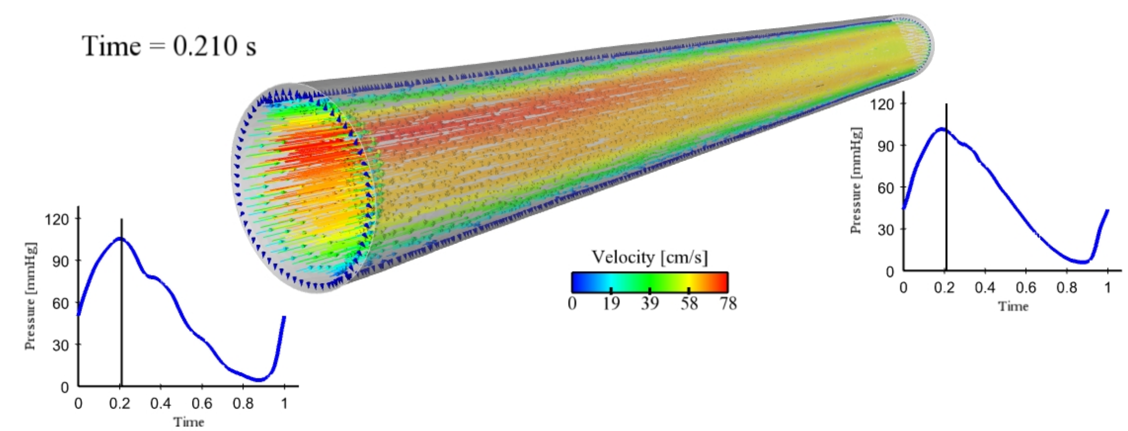


Simulation : C.Bertoglio

Experimental validation

- Silicon rubber aortic phantom



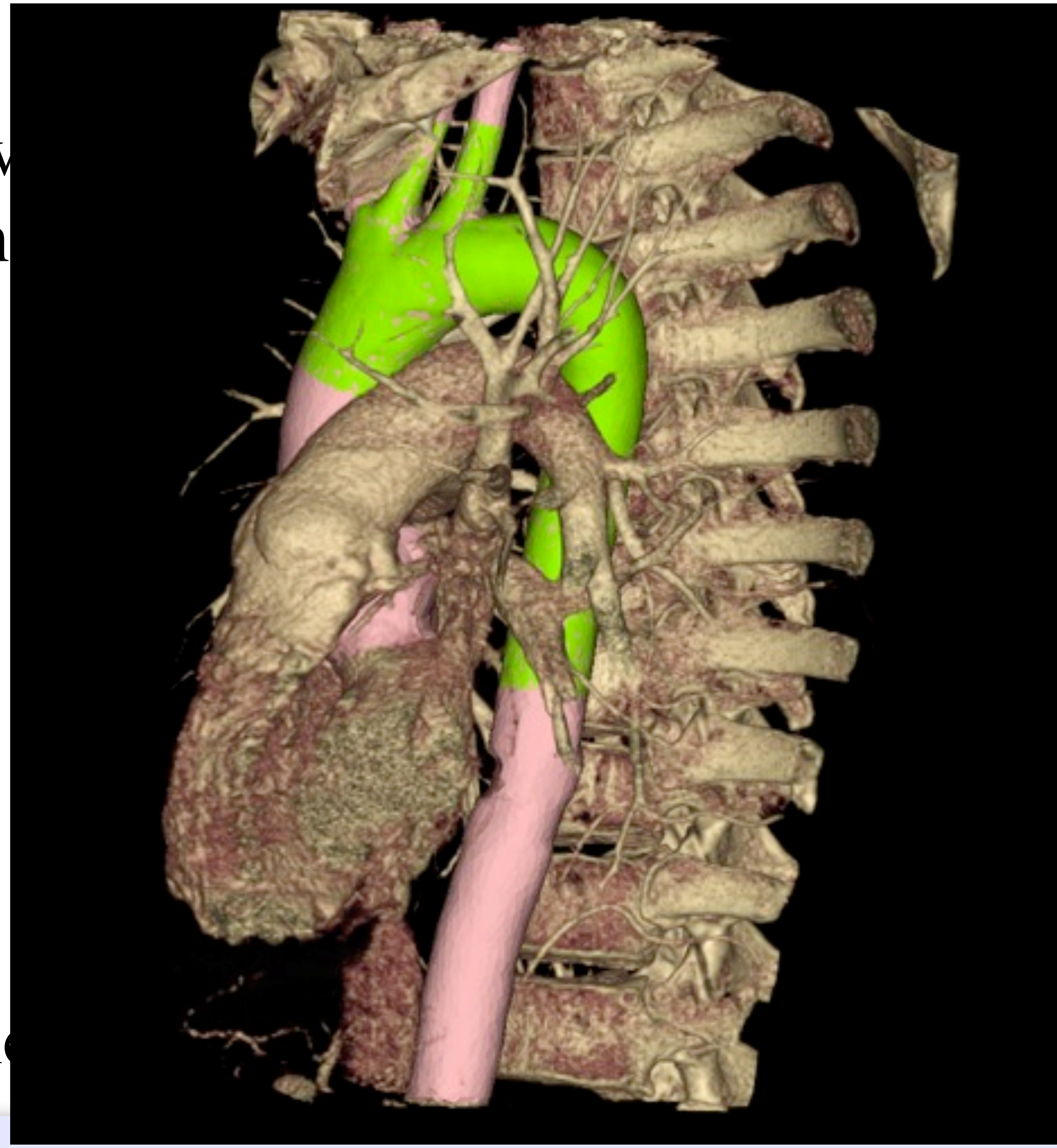


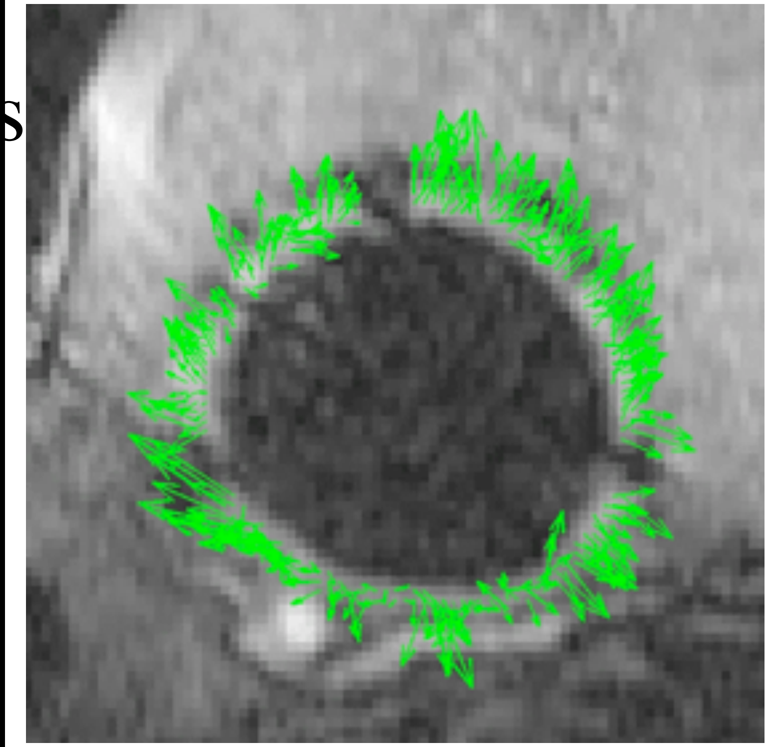
Data: Gaddum, Rutten, Beerbaum (KCL).
Segmentation and processing: Hose, Barber (Sheffield)
Simulation: Bertoglio, JFG (INRIA)
(euHeart project)

Blood flow in aorta

- Many w … s
but not th

- Typical
vessel :





Abdominal aorta
C. Taylor et al., Stanford

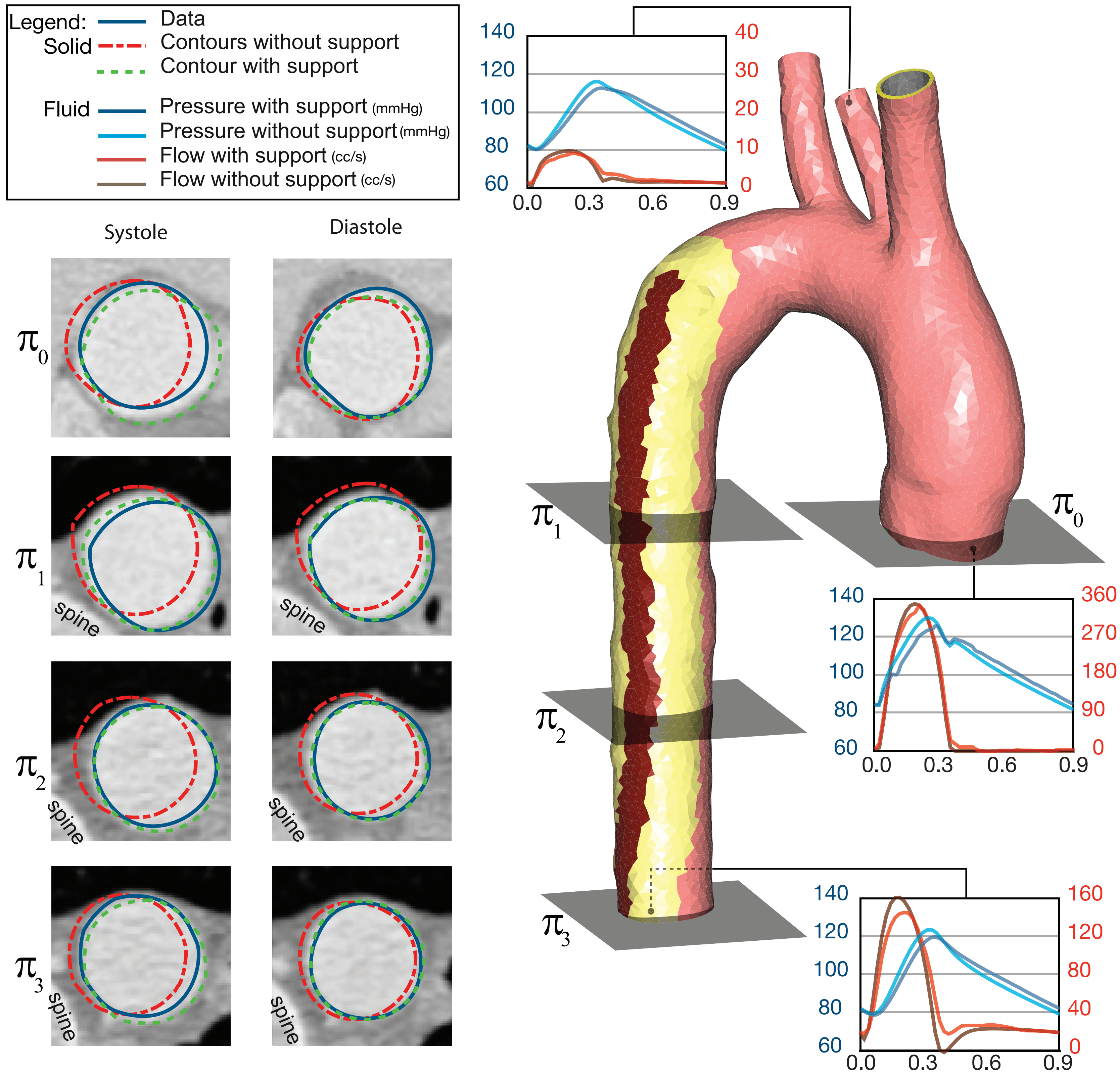
- A simpl … ternal tissues :

$$\boldsymbol{\sigma}_s \boldsymbol{n} = -k_s \boldsymbol{d} - c_s \frac{\partial \boldsymbol{d}}{\partial t}$$

Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)

Example 3: External tissue support

Modeling



Moireau, Xiao, Astorino, Figueroa, Chapelle, Taylor, JFG, (Biomech Model Mech 2011)

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- **Electrocardiograms (ECGs)**
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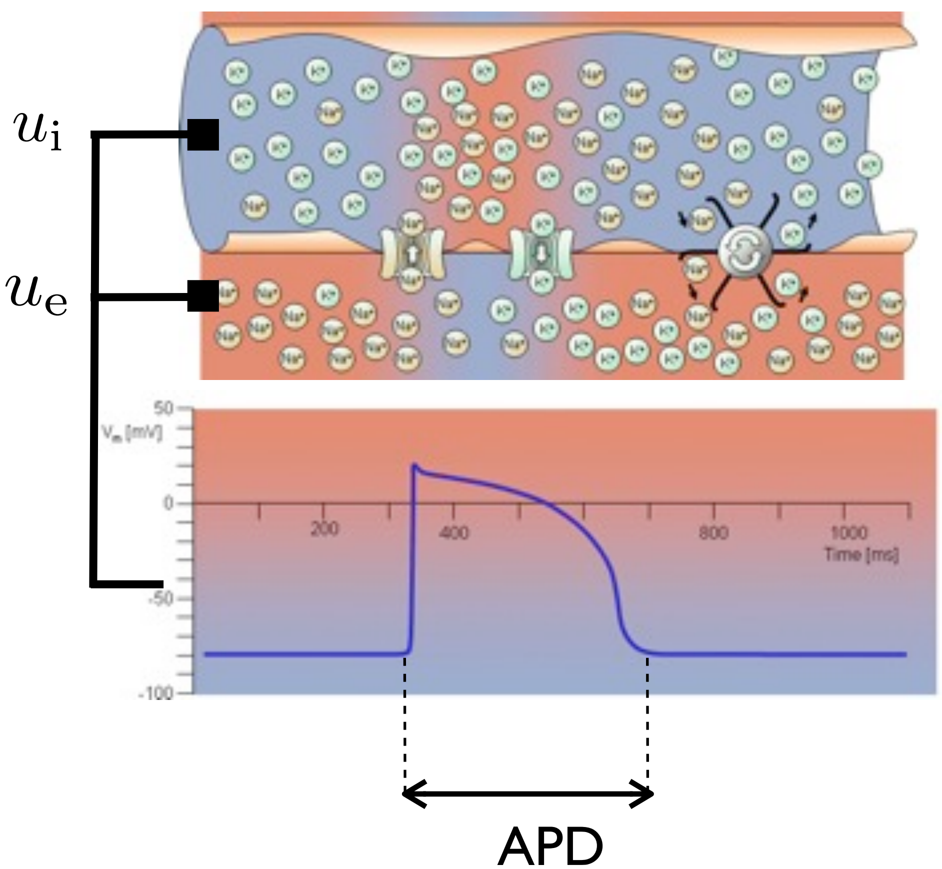
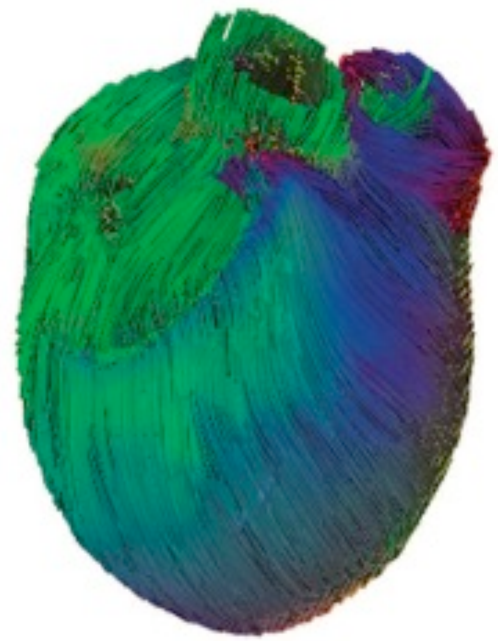
Electrocardiograms

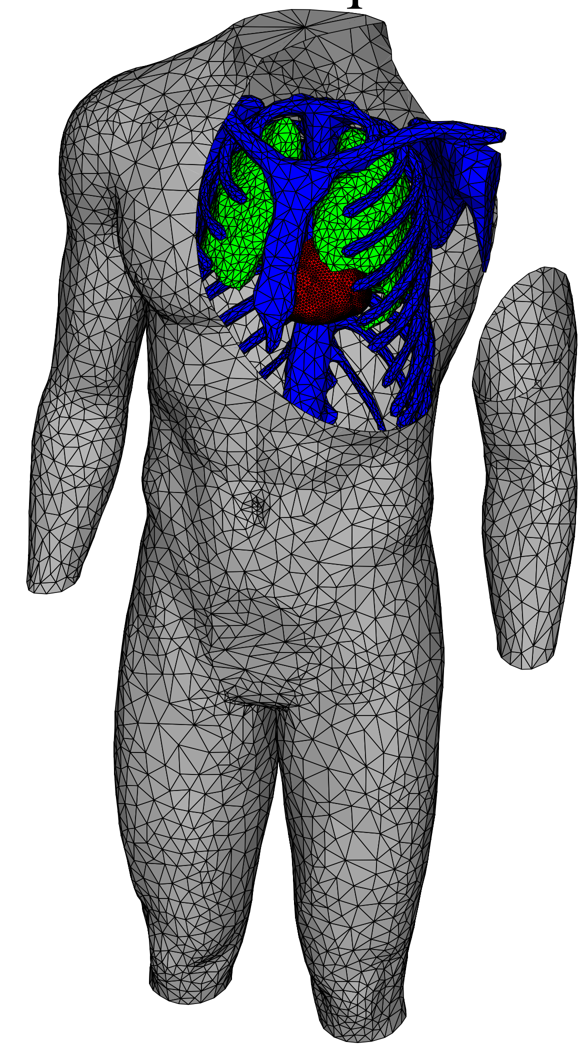
Cell scale:
Hodgkin-Huxley-like models

Myocardium:
bidomain models

Torso:
Poisson equation







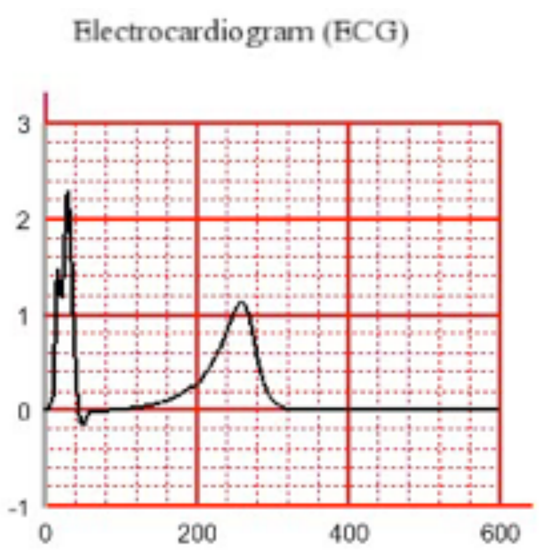
$$\begin{cases} C_\mathrm{m}\dfrac{\mathrm{d}V_\mathrm{m}}{\mathrm{d}t} + I_\mathrm{ion}(V_\mathrm{m},\boldsymbol{g}) = 0 \\ \dfrac{\mathrm{d}\boldsymbol{g}}{\mathrm{d}t} + G(V_\mathrm{m},\boldsymbol{g}) = \boldsymbol{0} \end{cases}$$

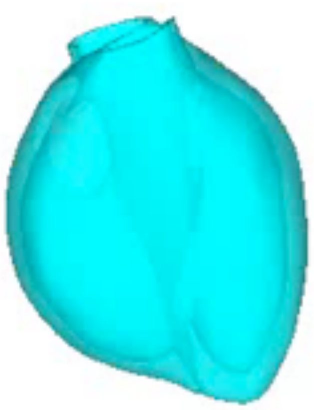
$$\begin{cases} A_\mathrm{m}\left(C_\mathrm{m}\dfrac{\partial V_\mathrm{m}}{\partial t} + I_\mathrm{ion}(V_\mathrm{m},\boldsymbol{g})\right) - \mathrm{div}(\boldsymbol{\sigma}_\mathrm{i}\boldsymbol{\nabla}u_\mathrm{i}) = A_\mathrm{m}I_\mathrm{app}, \\ \mathrm{div}(\boldsymbol{\sigma}_\mathrm{e}\boldsymbol{\nabla}u_\mathrm{e}) + \mathrm{div}(\boldsymbol{\sigma}_\mathrm{i}\boldsymbol{\nabla}u_\mathrm{i}) = 0 \\ \dfrac{\partial \boldsymbol{g}}{\partial t} + G(V_\mathrm{m},\boldsymbol{g}) = 0, \end{cases}$$

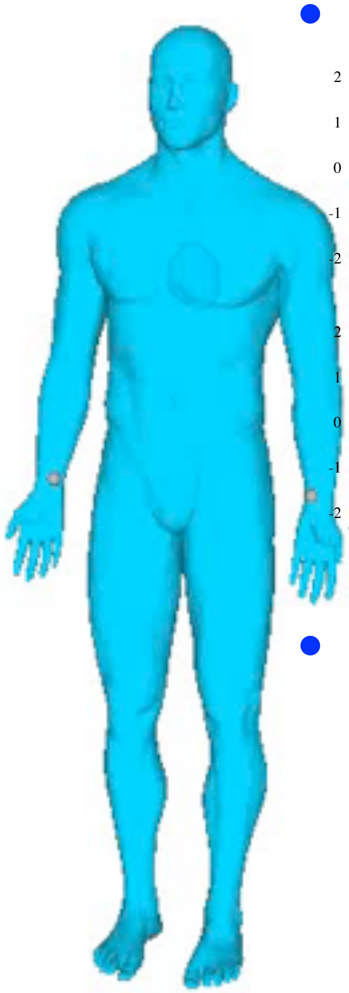
$$\begin{cases} \mathrm{div}(\boldsymbol{\sigma}_\mathrm{T}\boldsymbol{\nabla}u_\mathrm{T}) = 0 \\ + \textit{ transmission condition} \\ \textit{on the epicardium} \end{cases}$$

e.g.: Pullan et al. 05, Sundes et al. 06,...

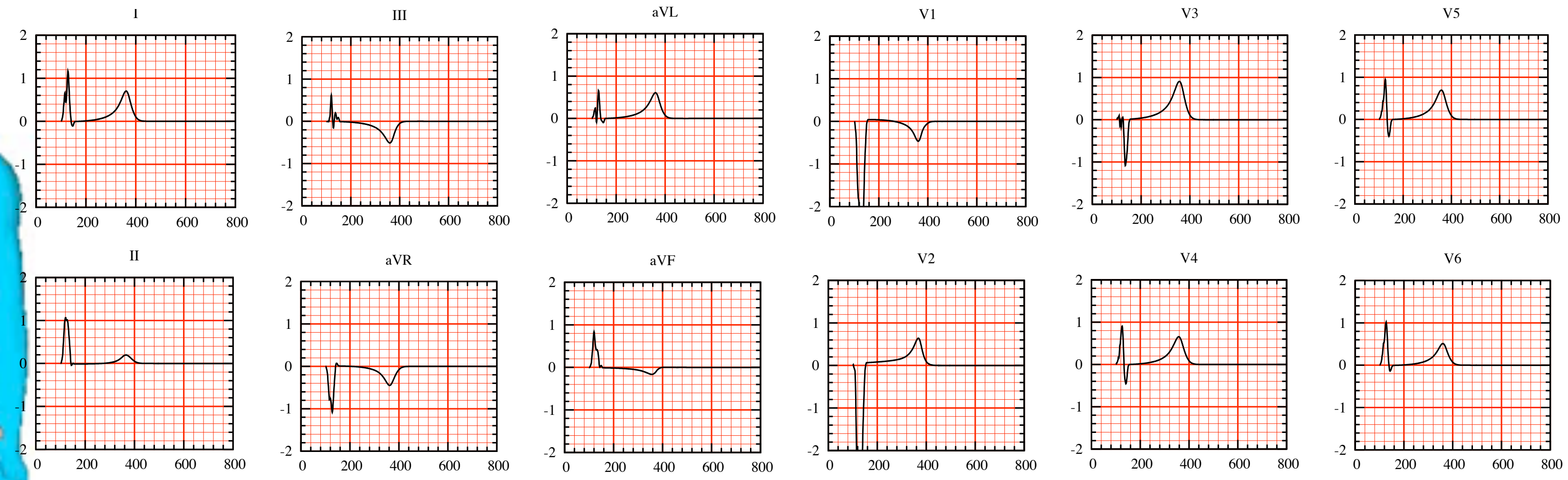
12-lead ECG



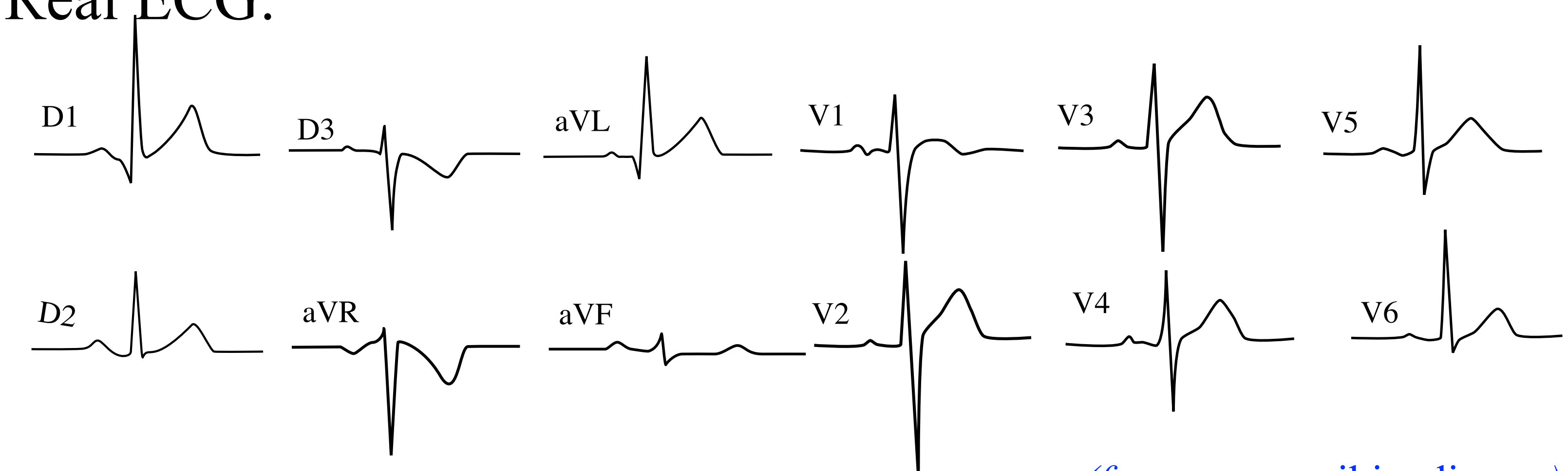




- Simulated ECG:



- Real ECG:

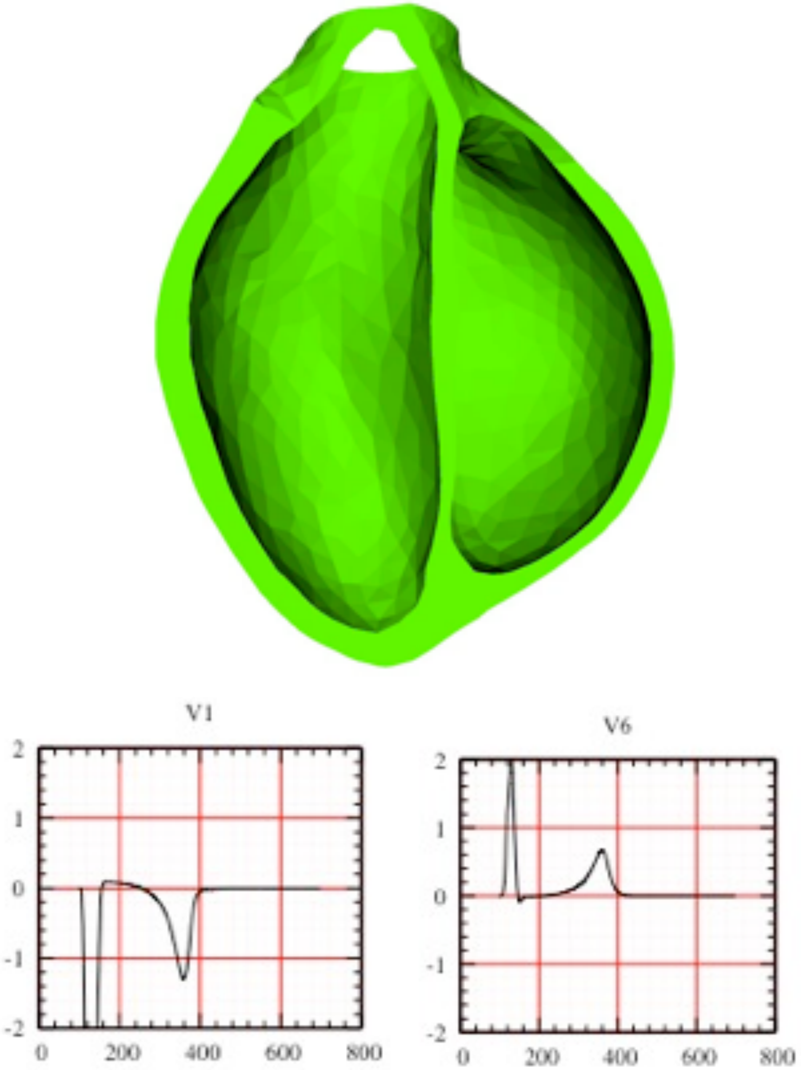


(from www.wikipedia.org*)*

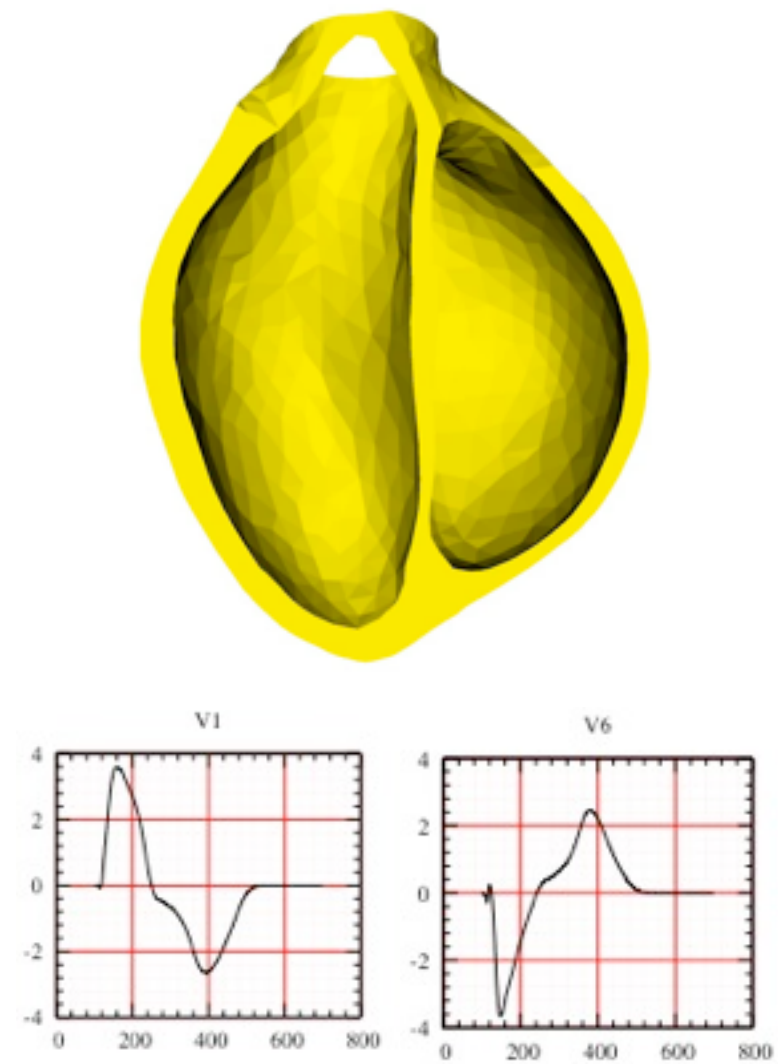
Boulakia, Cazeau, Fernández, JFG, Zemzemi, Annals Biomed Engng *2010*

Bundle branch blocks

Healthy case



Right bundle branch block



Chapelle, Fernández, JFG, Moireau, Sainte-Marie, Zemzemi, FIMH 2009

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Motivation: Blood flows

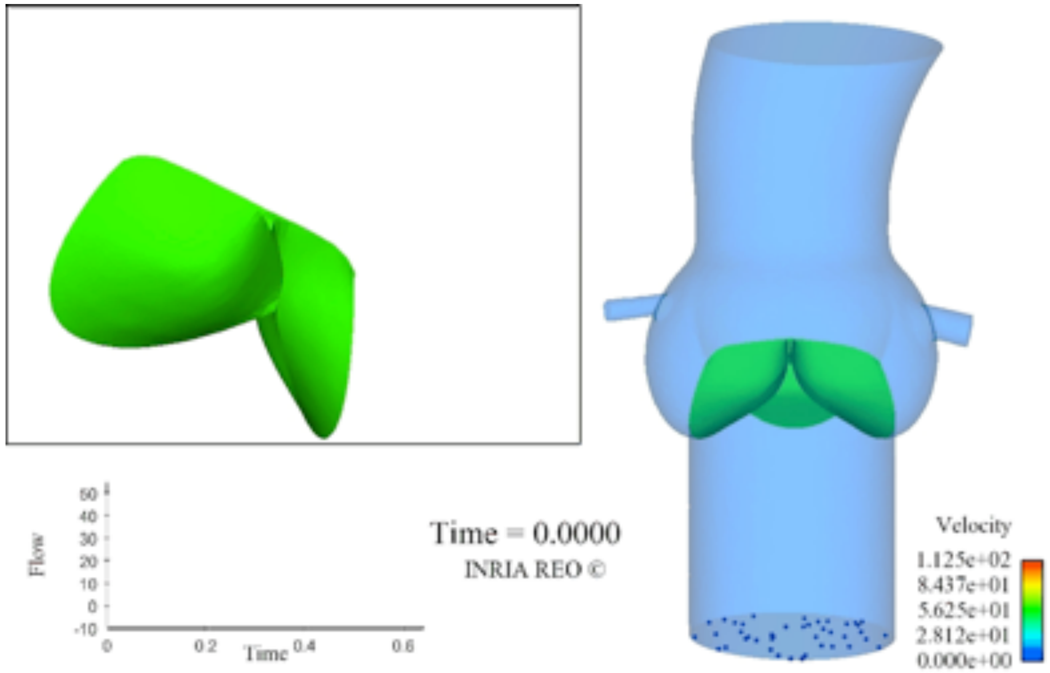
- **Output of interest**
  Ex: Pressure drop

- **Optimization**
  Ex: arterial by-pass optimization *(Quarteroni-Rozza, Sankaran-Marsden)*

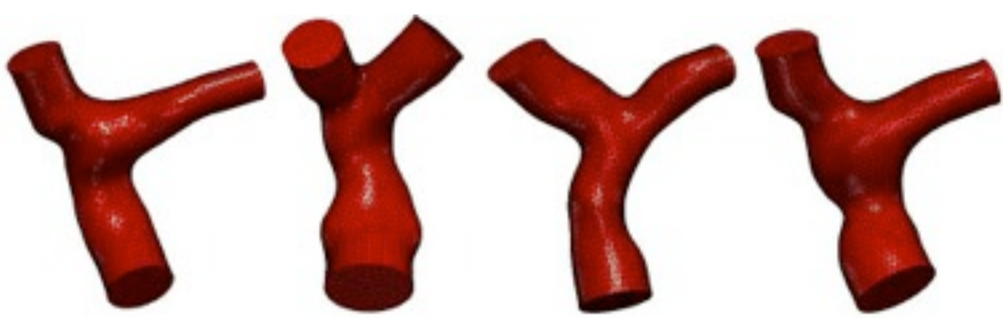
- **Rapid prototyping**
  Boundary conditions (Windkessel), constitutive law coefficients,...

- **Moderate variability**
  Examples: pulmonary arteries

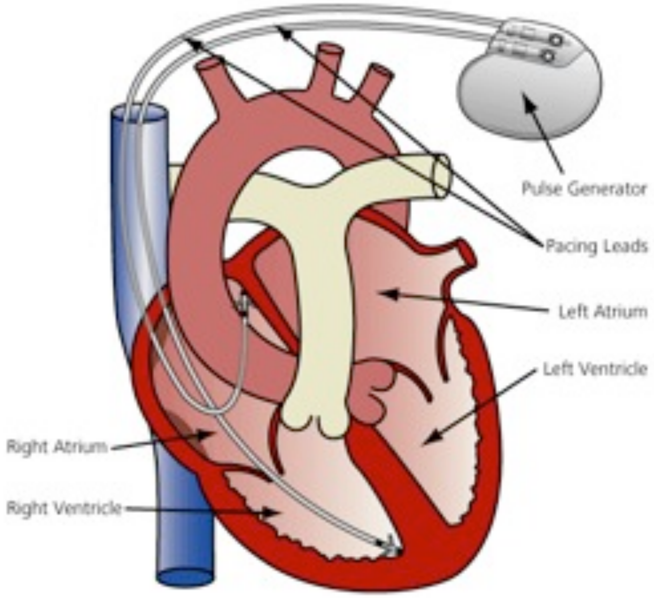
Aortic coarctation



Astorino, JFG, Pantz, Traoré, CMAME 2009

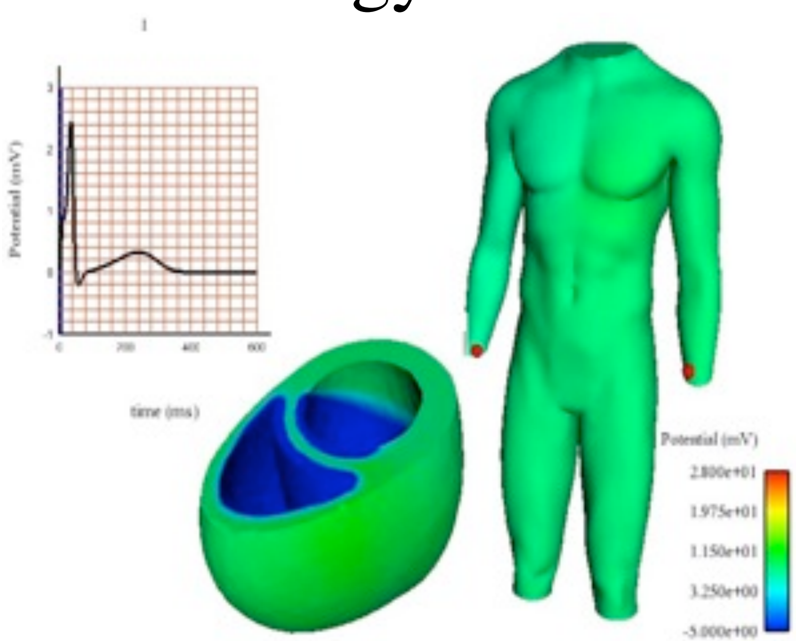


Motivation: electrophysiology



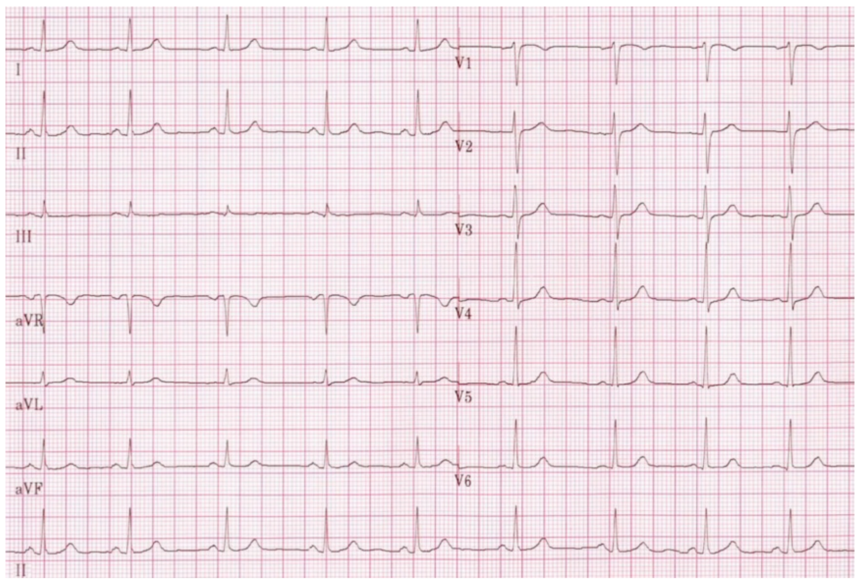
- **Optimization**
  Ex: optimize multi-sites pacing

- **Inverse problems**
  Electrocardiology



Forward

Inverse



Potential in heart and the torso

Electrocardiogram (ECG)

Reduced Order Modeling

Many options:

- ***Simplify the geometry and/or the physics***
  Examples:
  - 1D model for blood flows *(Formaggia, Peiró, ...)*
  - Eikonal model in electrophysiology *(Franzone, Sermesant, Frangi,...)*
- ***Keep the equations and the geometry, but reduce the approximation space***
  Examples:
  - Reduced Basis Method (*Patera, Maday, Quarteroni, Rozza,...)*
  - Proper Orthogonal Decomposition (POD), or Karhunen-Loève expansion *(Iollo, Farhat, Karniadakis, Kunisch, Gunzburger, Volkwein,...)*

POD in a nutshell

- Let $(\varphi_i)_{i=1..n}$ be a finite element basis

- Full Order Model (FOM): search for $u_h = \sum_{j=1}^{n} u_j \varphi_j$ such that:
$$\frac{d}{dt}(u_h, \varphi_i) + a(\theta; u_h, \varphi_i) = (f, \varphi_i), \forall i = 1..n$$

- Compute p "snapshots" (i.e. solutions at p time instants, *or* parameters θ):
$$S^1(u_1^1, \ldots, u_n^1), \ldots, S^p(u_1^p, \ldots, u_n^p)$$

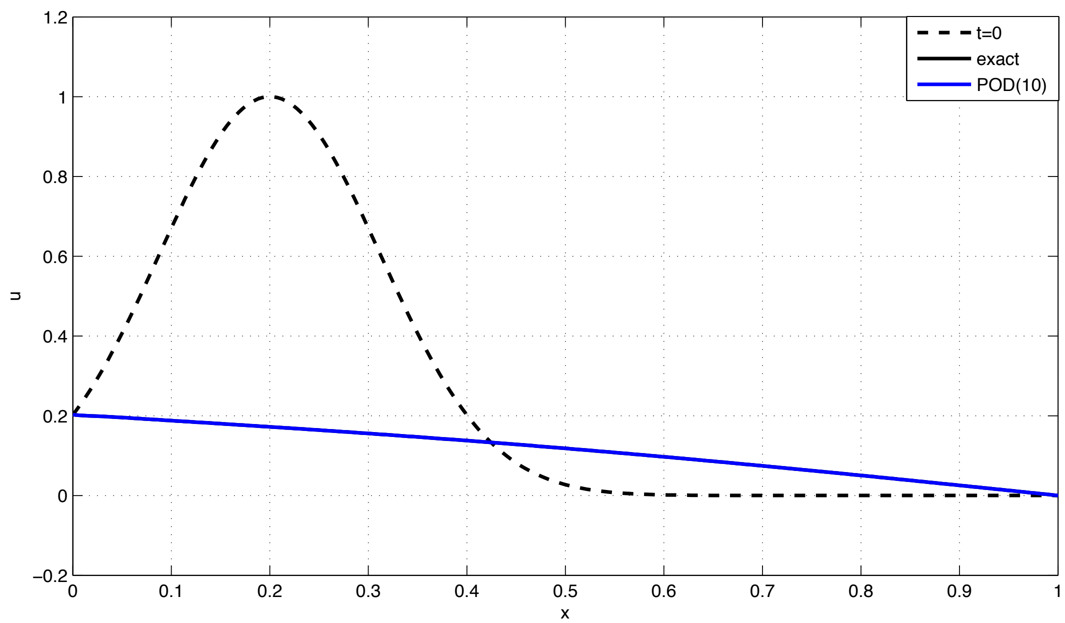
- Let S be the matrix whose columns are the S^i, $i = 1..p$.

- Singular Value Decomposition: $S = \Phi\, \Sigma\, \Psi^T$, with $\Sigma = diag(\sigma_1, \ldots, \sigma_p)$

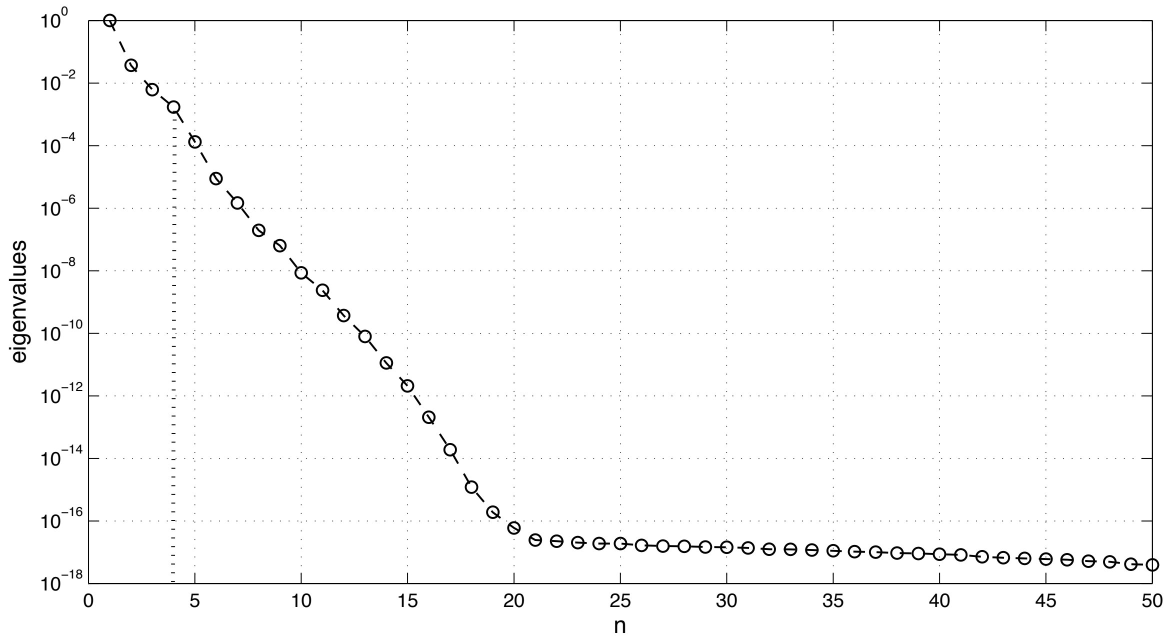
- $(\Phi_1, \ldots, \Phi_N)$: N columns of Φ corresponding to the N largest σ_i, $\mathbf{N \ll n}$

- Reduced Order Model (ROM): search for $U_h = \sum_{j=1}^{N} U_j \Phi_j$ such that:
$$\frac{d}{dt}(U_h, \Phi_i) + a(U_h, \Phi_i) = (f, \Phi_i), \forall i = 1..N$$

Example 1: $\dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = 0$



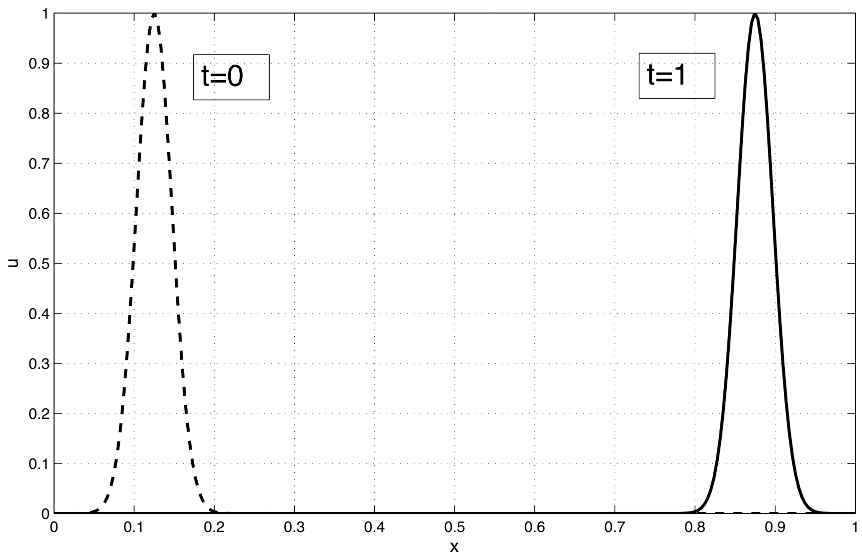
POD with 10 modes

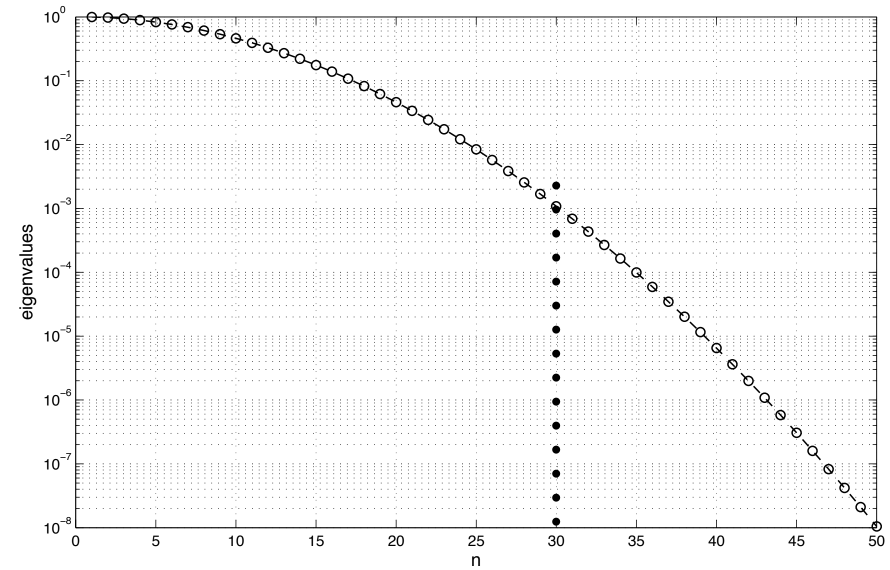


Eigenvalues

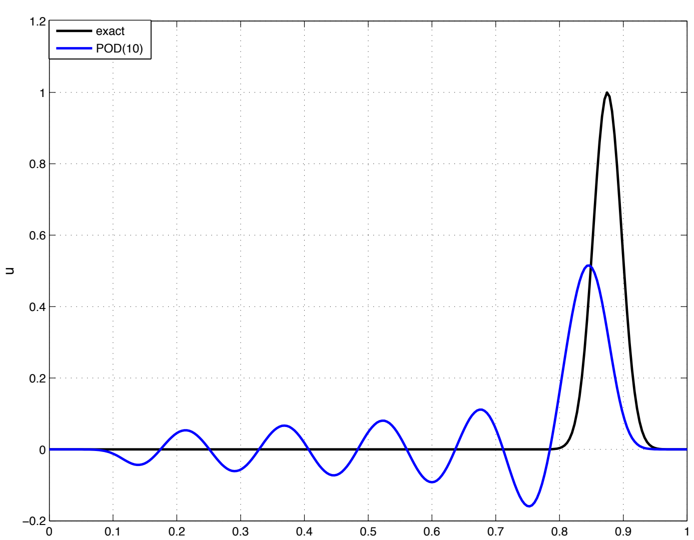
Simulation : D. Lombardi

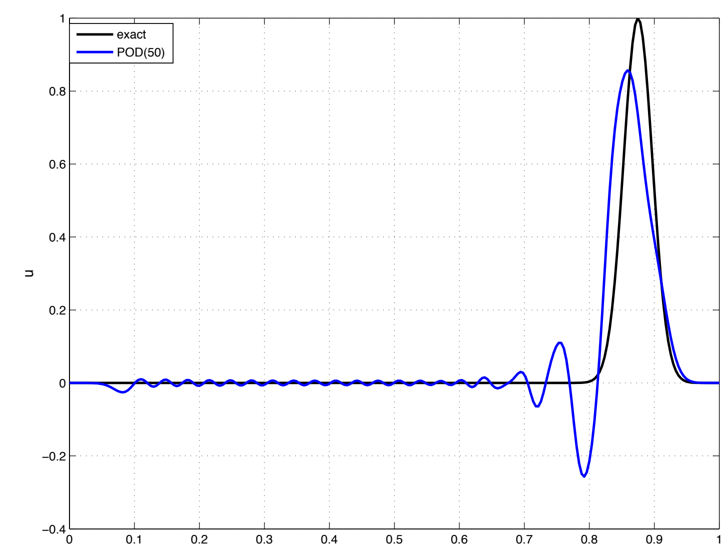
Example 2: $\frac{\partial u}{\partial t} + c\frac{\partial u}{\partial x} = 0$





Eigenvalues





POD with 10 modes

POD with 50 modes

Simulation : D. Lombardi

Basic idea

- Instead of

$$U_h = \sum_{j=1}^{N} U_j(t)\Phi_j(x)$$

- Try to work with something like :

$$U_h = \sum_{j=1}^{N} U_j\Phi_j(x,t)$$

- **Question** : how to propagate the modes ?

Step 1: Semi Classical Signal Analysis (SCSA)

Laleg, Crépeau, Sorine (2007 & 2012)

- Let *u(x)* be a non-negative signal and the "scattering" operator

$$\mathcal{L}_\chi(u)\phi = -\Delta\phi - \chi u\phi$$

- Solve the eigenvalue problem

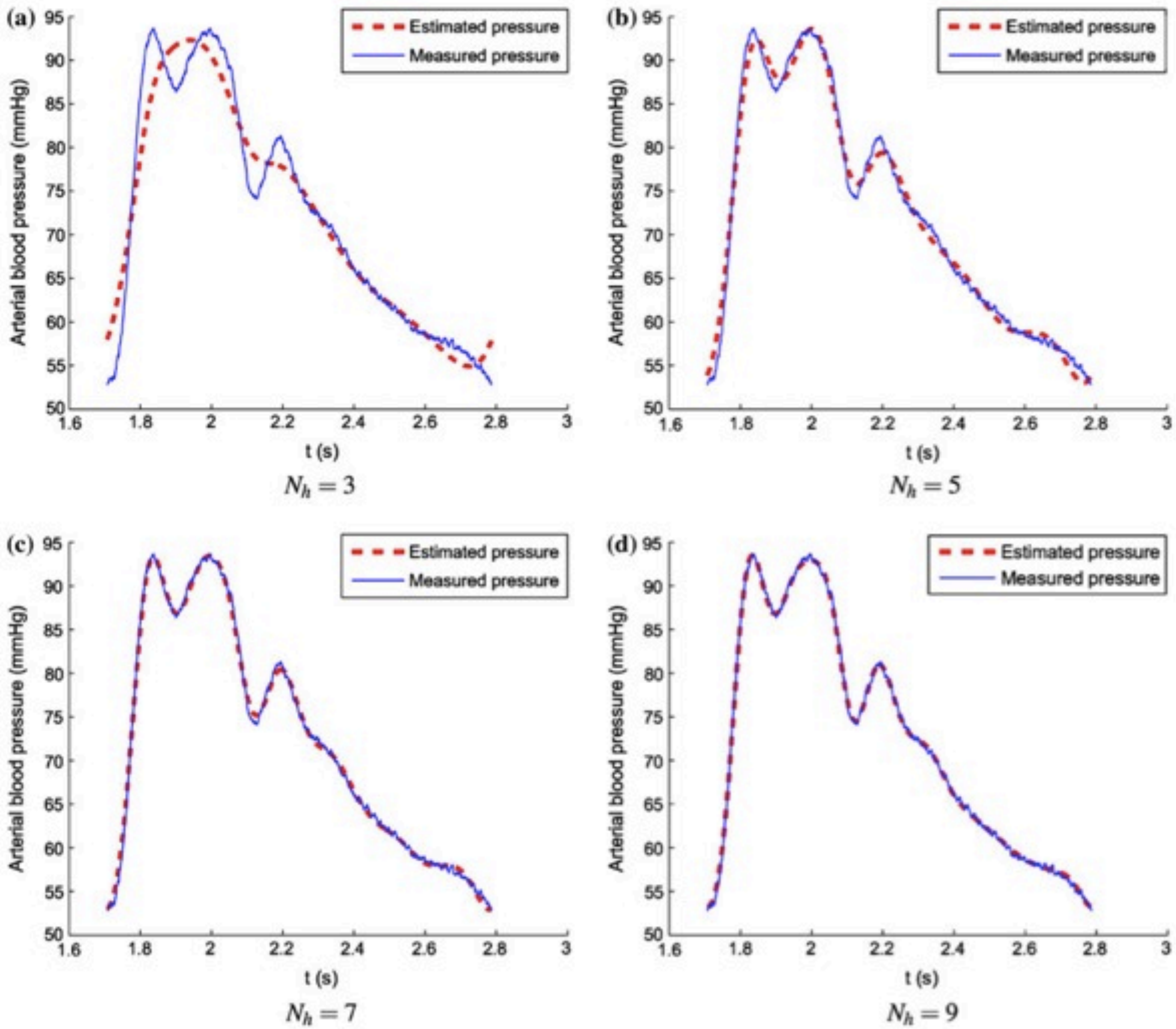
$$\mathcal{L}_\chi(u)\phi = \lambda\phi$$

- And approximate *u(x)* by the Deift-Trubowitz formula

$$\boxed{\tilde{u}(x) = \chi^{-1}\sum_{m=1}^{N_-}\kappa_m\phi_m^2}$$

with $\kappa_m = \sqrt{-\lambda_m}$ where λ_m are the negative eigenvalues

- Choose $\chi > 0$ to achieve a given accuracy
- Remark: can be exact for "reflectionless" potentials



Reconstruction of aortic pressure

From: *Laleg, Médigue, Papelier, Crépeau, et al. (2010)*

Lax pair in a nutshell

- Let $\mathcal{L}(t)$ be a self-adjoint operator in a Hilbert space
- If an operator $\mathcal{M}(t)$ is such that

$$\partial_t \mathcal{L} + \mathcal{L}\mathcal{M} - \mathcal{M}\mathcal{L} = 0$$

- If $(\lambda_k(t), \phi_k(t))$ is an eigenpair of $\mathcal{L}(t)$ then
  - $\partial_t \lambda_k = 0$
  - $\partial_t \phi_k(t) = \mathcal{M}(t)\phi_k(t)$

Lax pair in a nutshell

- Example: Korteweg de Vries equation:

$$\partial_t u + 6u\partial_x u + \partial_x^3 u = 0$$

- Lax pair:

$$\mathcal{L}(u)v = -\partial_x^2 v - uv$$

$$\mathcal{M}(u)v = 4\partial_x^3 v + 6u\partial_x v + 3v\partial_x u$$

- Example: 1-soliton: if $u_0(x) = \frac{\kappa_1^2}{2}\phi_1^2(\kappa_1 x)$

  then: $u(x,t) = \frac{\kappa_1^2}{2}\phi_1^2(\kappa_1(x - \kappa_1^2 t))$

with: $\phi_1(x) = \mathrm{sech(x)}$

Goal:

- Consider a general nonlinear evolution PDE :

$$\begin{cases} \partial_t u &= F(u) \\ u(0) &= u_0 \end{cases}$$

- For example: $F(u) = \Delta u + u(1-u)$

- Use the SCSA and the Lax pair to define a reduced order model

- Difficulty: the Lax pair are not known in general

Step 2: Approximation of Lax Pair (ALP)

Proposition: let u be solution to

$$\begin{cases} \partial_t u & = & F(u) \\ u(0) & = & u_0 \end{cases}$$

- For $m \in \{1, \ldots, N_M\}$, let $\lambda_m(t)$ be an eigenvalue of $\mathcal{L}_\chi(u(x,t))$ and $\psi_m(x,t)$ an associated eigenfunction, normalized in $L^2(\Omega)$
- Assume there exists an operator $\mathcal{M}(u)$ such that

$$\partial_t \psi_m = \mathcal{M}(u)\psi_m.$$

- The evolution of λ_m is governed by

$$\boxed{\partial_t \lambda_m = -\chi \langle F(u)\psi_m, \psi_m\rangle}$$

- The evolution of ψ_m satifies, for $p \in \{1, \ldots, N_M\}$,

$$\boxed{\langle \partial_t \psi_m, \psi_p\rangle = M_{mp}(u)}$$

with $\begin{cases} M_{mp}(u) & = & \dfrac{\chi}{\lambda_p - \lambda_m}\langle F(u)\psi_m, \psi_p\rangle, \quad \text{if } p \neq m \text{ and } \lambda_p \neq \lambda_m, \\ M_{mp}(u) & = & 0, \quad \text{if } p = m \text{ or } \lambda_p = \lambda_m. \end{cases}$

Step 3: Approximated "inverse scattering"

- $(\phi_m, \lambda_m)_{m=1..N_-}$: eigenfunctions/values of $\mathcal{L}_\chi(u)$

- We look for an approximation of u:

$$\tilde{u} = \sum_{k=1}^{N_-} \alpha_k \phi_k^2$$

- Inserting this expression in

$$\langle \mathcal{L}_\chi(\tilde{u})\phi_m, \phi_p \rangle = \lambda_m \langle \phi_m, \phi_p \rangle$$

- We find

$$\boxed{\sum_{k=1}^{N_-} \alpha_k \langle \phi_k^2, \phi_m^2 \rangle = -\frac{1}{\chi} \left(\lambda_m + \langle \Delta \phi_m, \phi_m \rangle \right).}$$

which gives the α_k.

Summary: the ALP algorithm

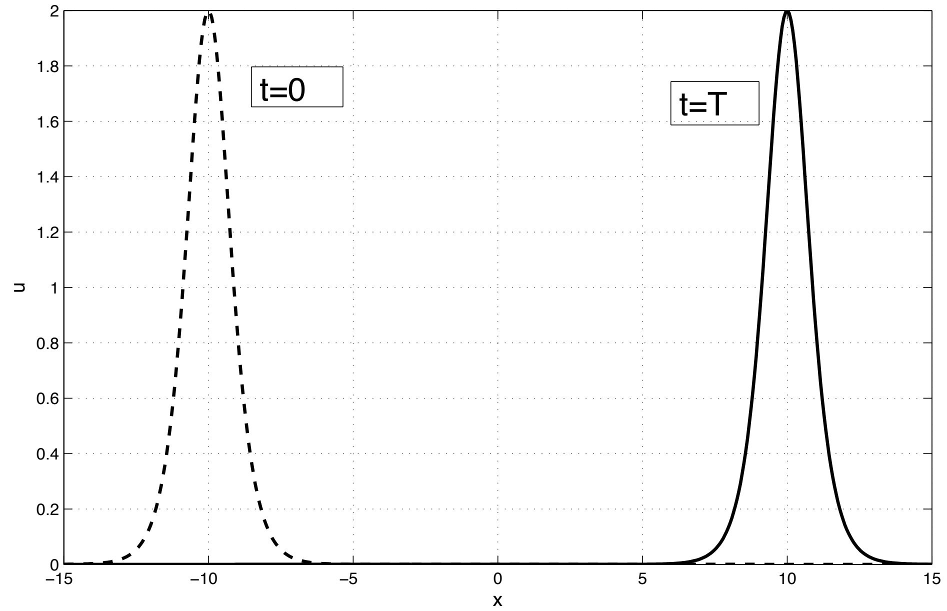
Initialisation (SCSA of u_0): compute N_M eigenmodes ψ_m.
The N_- first are denoted ϕ_m and

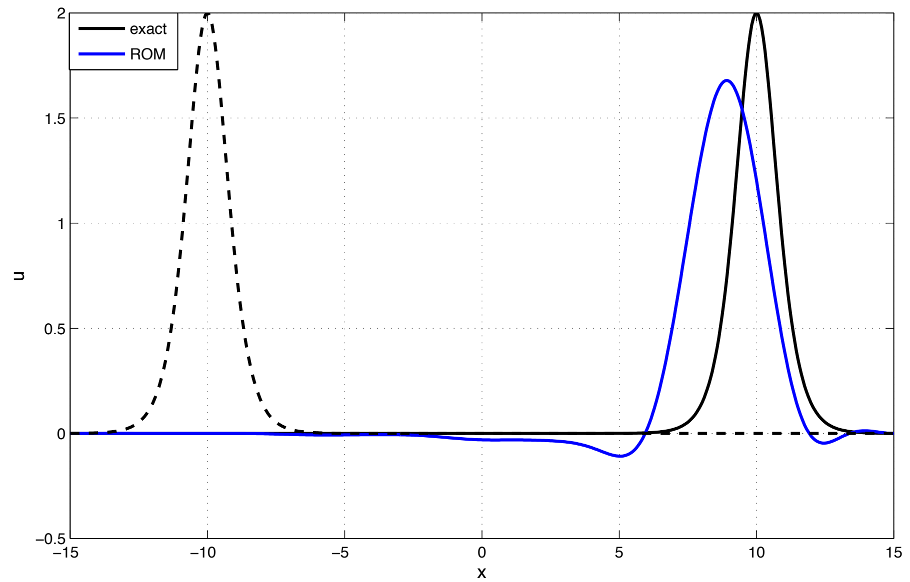
$$u_0(x) \approx \chi^{-1} \sum_{m=0}^{N_-} \kappa_m \phi_m(x)$$

1. Compute the matrix $M(u^n)$ approximating the operator $\mathcal{M}(u(t^n))$
2. Compute the eigenvalues λ_m^{n+1}: $\lambda_m^{n+1} = \lambda_m^n - \delta t \langle F(u^n)\psi_m^n, \psi_m^n \rangle$
3. Compute the eigenfunctions ψ_m^{n+1}:
   $\psi_m^{n+1} = \sum_{p=1}^{N_M} \left[I + \delta t M(u^n) + \frac{\delta t^2}{2} M(u^n)^2 \right]_{mp} \psi_p^n$
4. Solve for α_p^{n+1}: $\sum_{p=1}^{N_-} \alpha_p^{n+1} \langle \phi_p^2, \phi_m^2 \rangle = -\frac{1}{\chi} \left(\lambda_p^{n+1} + \langle \Delta\phi_m^{n+1}, \phi_m^{n+1} \rangle \right)$.
5. Compute u^{n+1} with $u^{n+1} = \sum_{p=1}^{N_-} \alpha_p^{n+1} \left(\phi_p^{n+1} \right)^2$

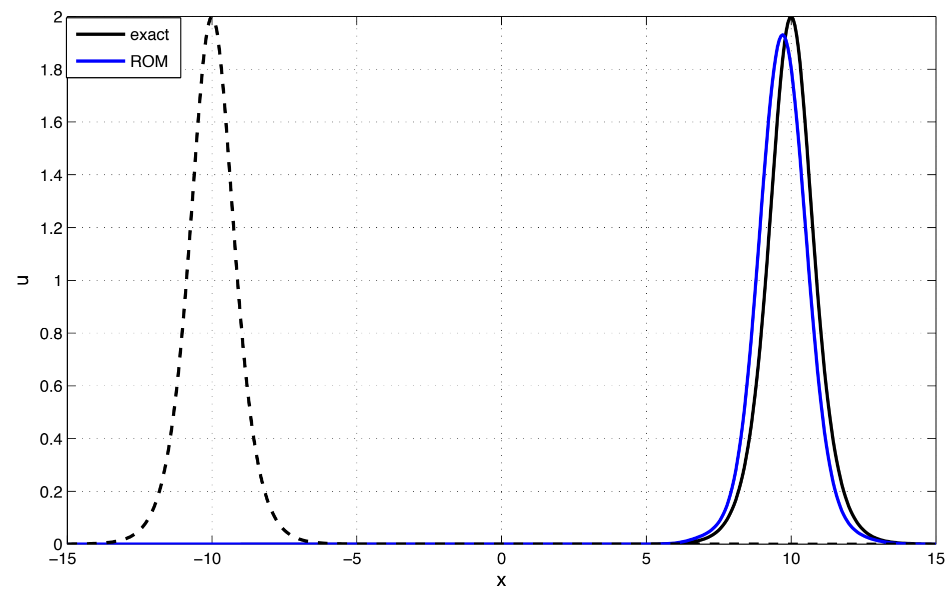
Numerical test cases

- Korteweg-de Vries:
  - ★ one-soliton (analytically known)
  - ★ only one negative eigenvalue





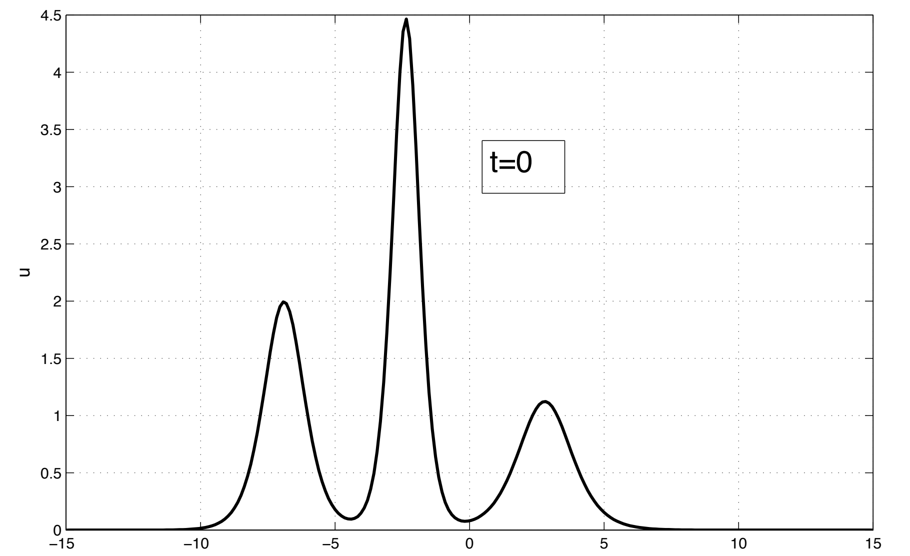
10 modes

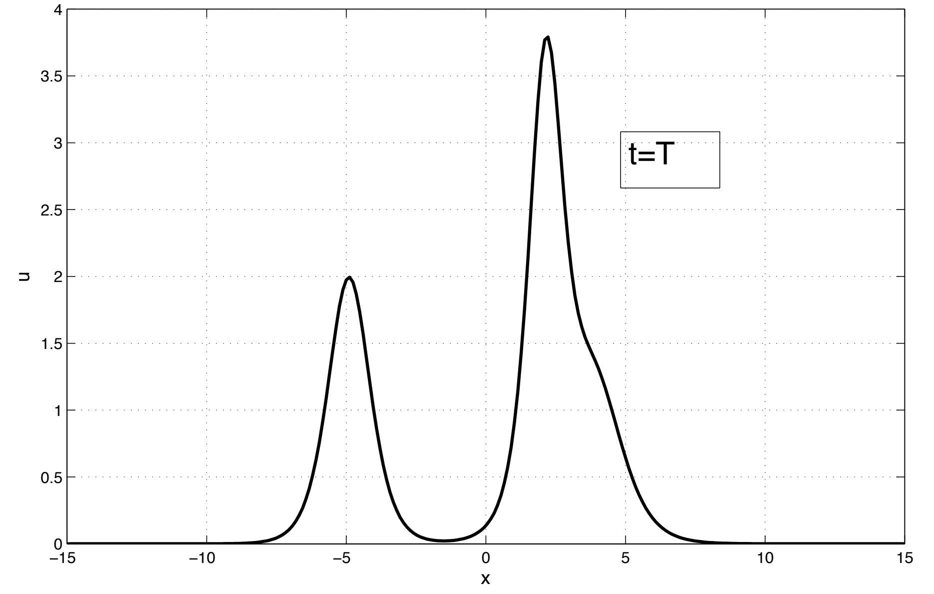


25 modes

Simulation : D. Lombardi

- Korteweg-de Vries:
  - ★ three-soliton
  - ★ three negative eigenvalues

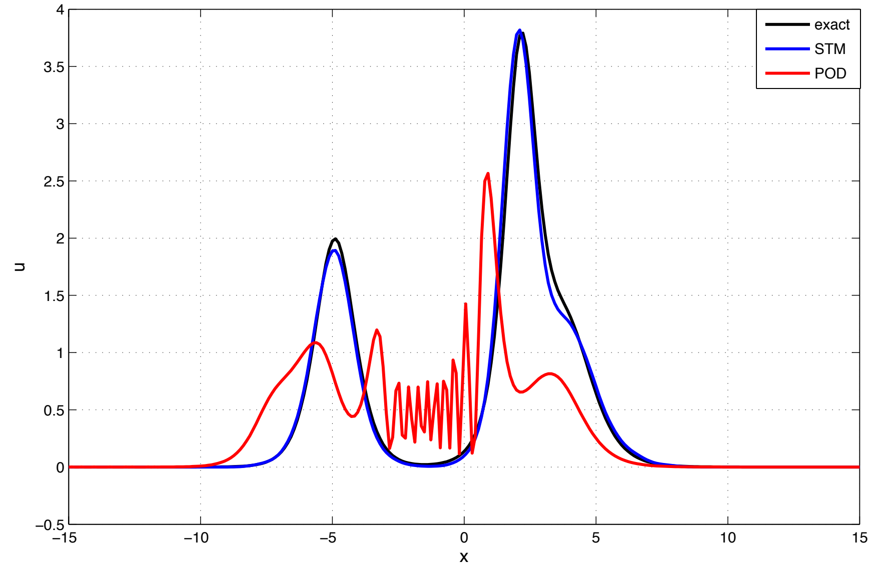




- Comparison of:

★ projection of the analytic solution on 20 POD modes built of snapshots obtained from the half first time history

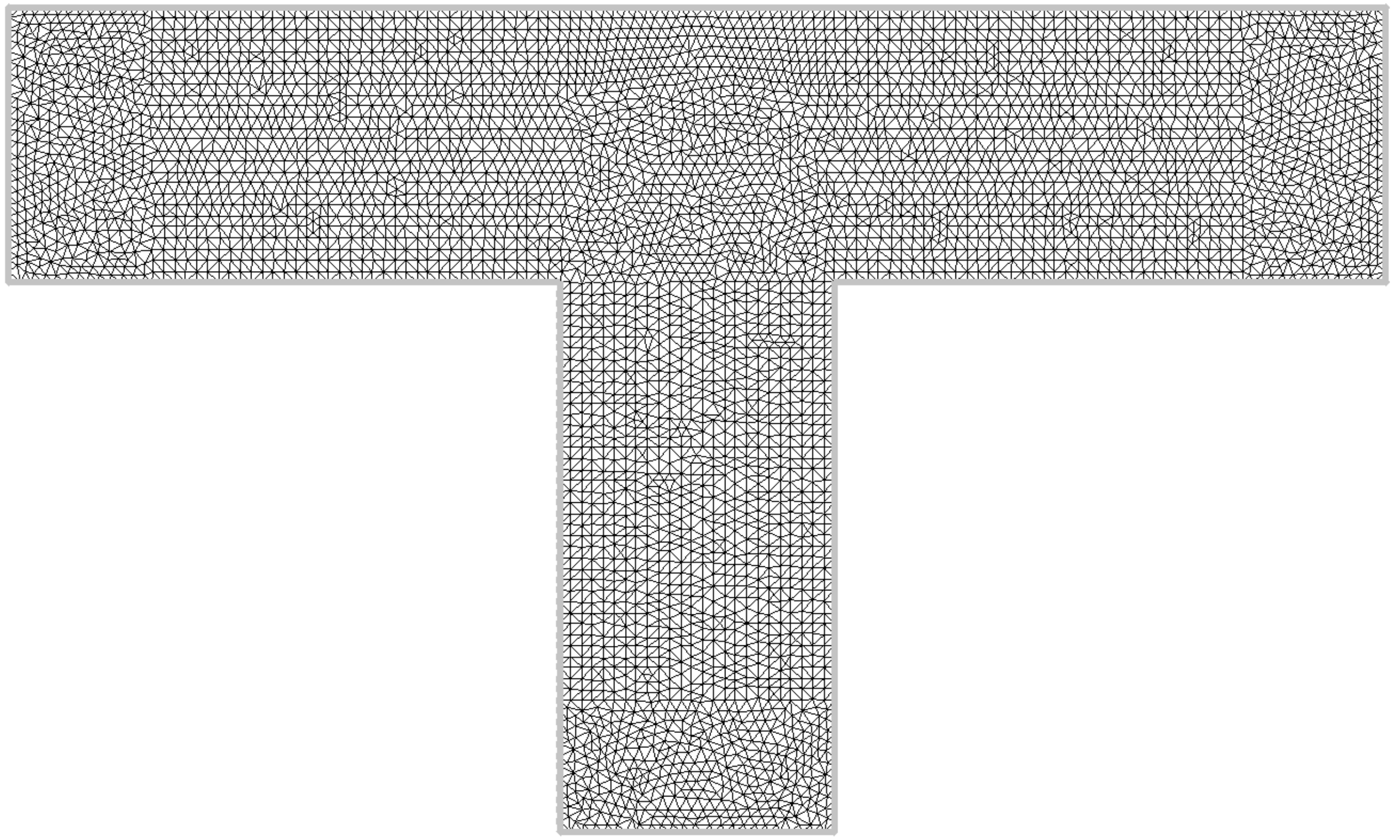
★ ALP integration with 20 modes

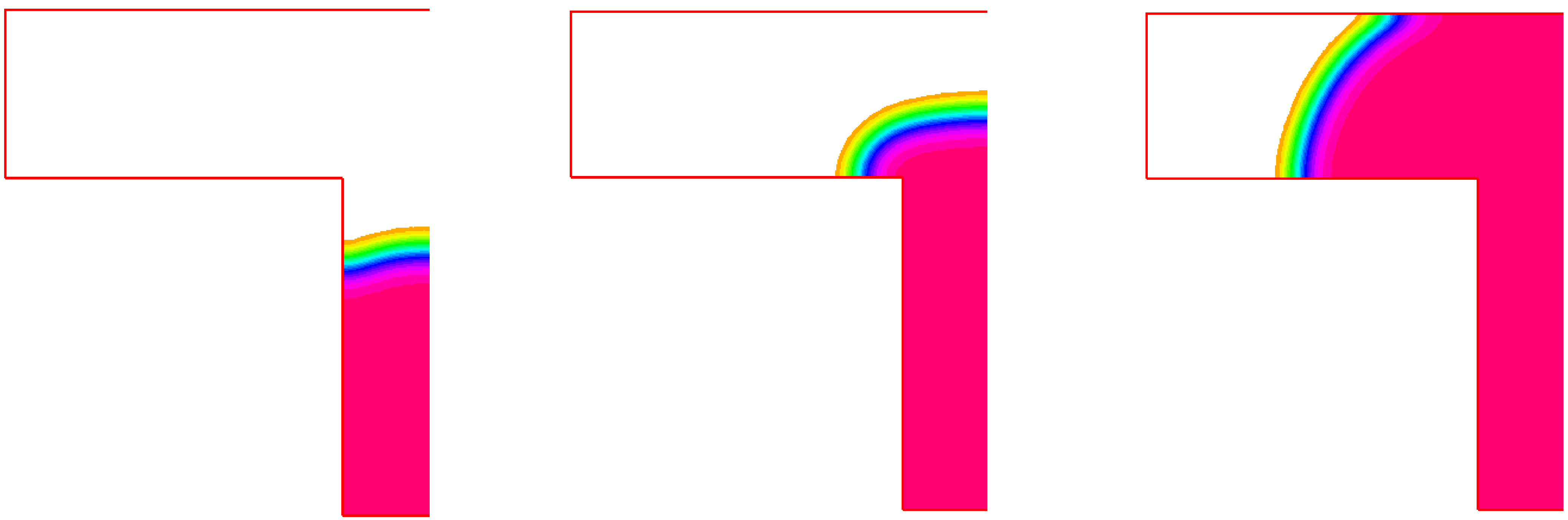


Simulation : D. Lombardi

- Fisher-Kolmogorov-Petrovski-Piskunov:

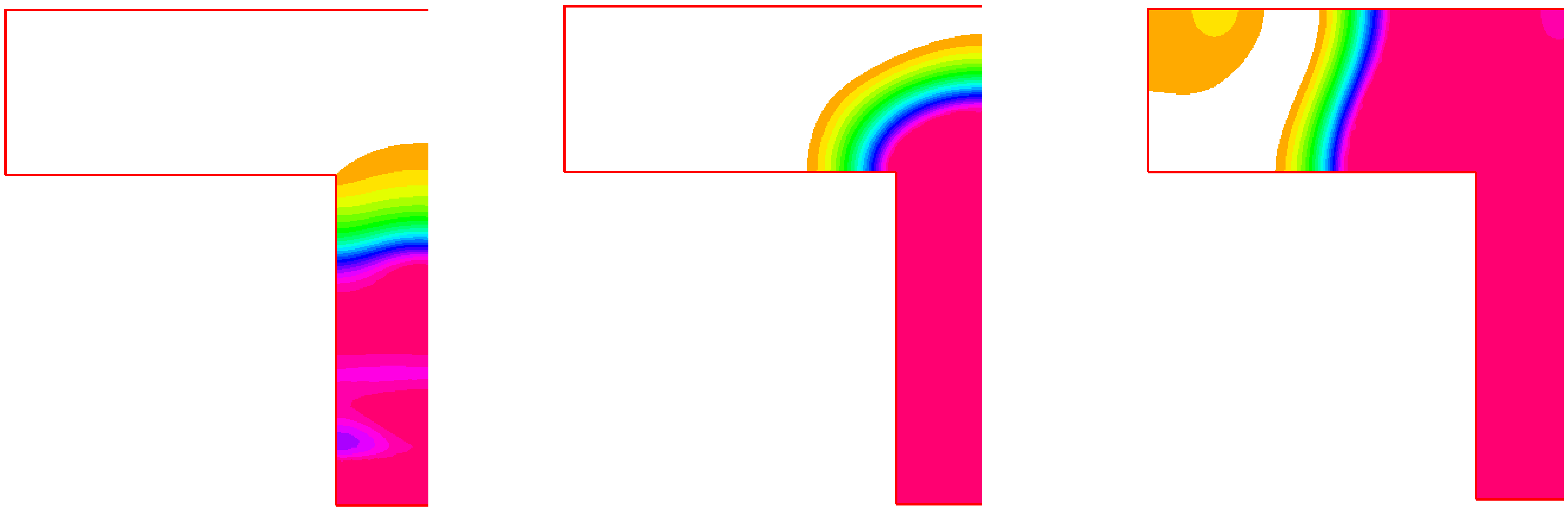
$$\frac{\partial u}{\partial t} - \Delta u = u(1-u)$$





- P1 Finite-element:
  - ⋆ 9000 dof
  - ⋆ Time step 2.5e-4
- Challenges:
  - ⋆ Sharp front, splitting in two directions
  - ⋆ No precomputed solutions : POD cannot work

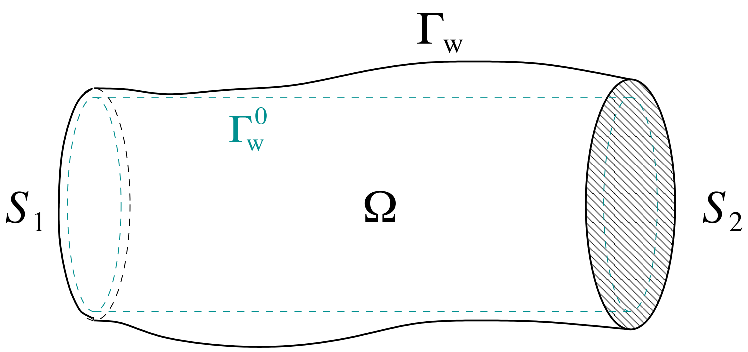
Simulation : D. Lombardi



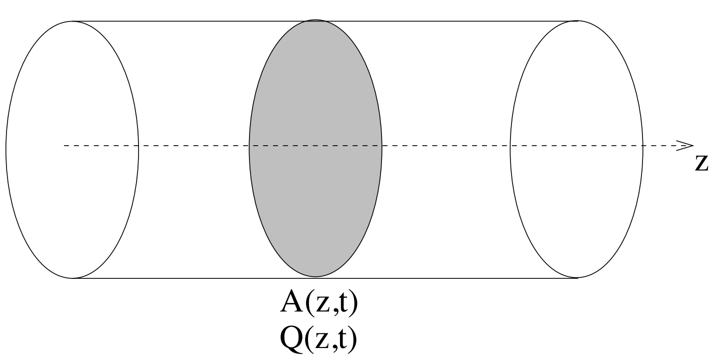
- ALP
    - ★ 6 modes for the solution, 25 modes for the operator
    - ★ Time step 2.5e-3 (10 times larger than FEM)

Simulation : D. Lombardi

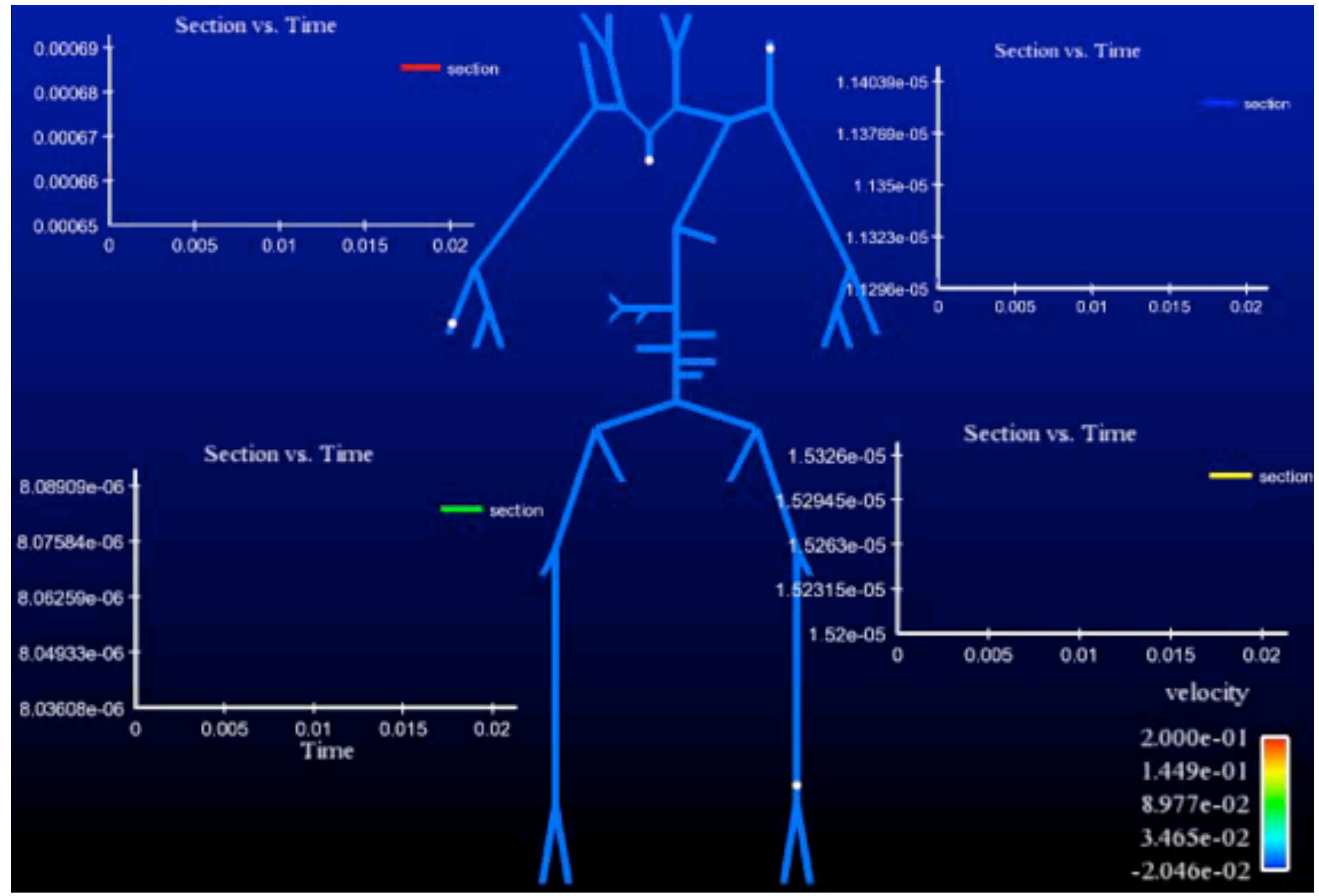
In progress....



velocity and pressure



flow rate and mean pressure



55 main arteries

(D. Lombardi)

Perspectives

- Many things to do
  - ★ Adaptation of the number of modes
  - ★ Different operators for the scattering step
  - ★ Role of positive eigenvalues
  - ★ Error estimator
  - ★ Analysis
  - ★ ...
- Applications (in progress)
  - ★ systemic network of arteries *(Damiano Lombardi)*
  - ★ cardiac electrophysiology *(Elisa Schenone)*

Inria Research Report 8137 available on HAL
http://hal.inria.fr/hal-00752810

- Remark:

$$\int_\Omega (\partial_t \psi_m)^2 \approx \sum_{n,l=1}^{N_M} M_{ml}(u) M_{mn}(u) \langle \psi_n, \psi_l \rangle = \sum_{n=1}^{N_M} M_{mn}(u)^2$$

- Thus, the Frobenius norm of *M(u)* can be used as an estimator of the approximation of the dynamics